3.1.1 Are they equivalent?

Equivalent Expressions

In this chapter you will look at how to rewrite expressions and equations into equivalent forms that will make them more useful. In this lesson, you will begin by identifying equivalent expressions and then work on developing algebraic strategies to show that they are equivalent.

3-1. Consider the tile pattern at right.

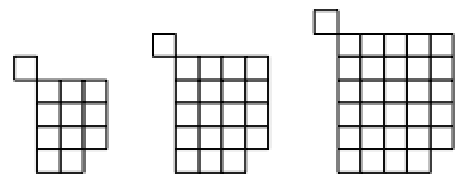


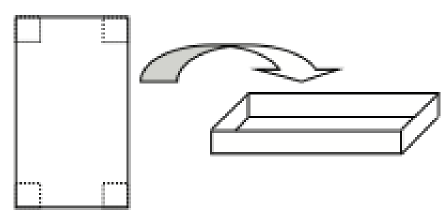
Figure 1 Figure 2 Figure 3

a. Work with your team to describe what the 100th figure would look like. Then find as many different expressions as you can for the area (the number of tiles) in Figure x. Use algebra to justify that all of your expressions are equivalent.

b. What information about the pattern is given by various parts of your different expressions?

c. Write and solve an equation to determine which figure number has 72 tiles. Do you get different results depending upon which expression you choose to use? Explain.

3-2. Jill and Terrell were looking back at their work on problem 1-53 in Lesson 1.2.1. They had come up with two different expressions for the volume of a paper box made from cutting out squares of dimensions x centimeters by x centimeters. Jill's expression was $(15 - 2x)(20 - 2x)x$, and Terrell's expression was $4x^3 - 70x^2 + 300x$.



a. Are Jill's and Terrell's expressions equivalent? Justify your answer.

b. If you have not done so already, find an algebraic method to decide whether their expressions are equivalent. Be ready to share your strategy.

c. Gary joined in on their conversation. He had another expression: $(15 - 2x)(10 - x)2x$. Use a strategy from part (b) to decide whether his expression for the volume is equivalent to Jill's or Terrell's. Be prepared to share your ideas with the class.

3-3. For each of the following expressions, find at least three equivalent expressions. Be sure to justify how you know they are equivalent.

a. $(x + 3)^2 - 4$

b. $(2a^2b^3)3$

c. $m^2n^5 \cdot mn^4$

d. $\frac{(x+1)(2x-1)}{x+2}$

3-4. LEARNING LOG

What does it mean for two expressions to be equivalent? How can you tell if two expressions are equivalent? Answer these questions in your Learning Log. Be sure to include examples to illustrate your ideas. Title this entry "Equivalent Expressions" and label it with today's date.

3-5. For each of the following expressions, find at least three equivalent expressions. Which do you consider to be the simplest? Help (Html5)⇔Help (Java)

a. $(2x-3)^2+5$

b. $(\frac{3x^2y}{x^3})^4$

3-6. Match each expression on the left with its equivalent expressions on the right. Assume that all variables represent positive values. Be sure to justify how you know each pair is equivalent. Help (Html5)⇔Help (Java)

a. $\sqrt{4x^2y^4}$	1. $2x\sqrt{y}$
b. $\sqrt{8x^2y}$	2. $2y\sqrt{2x}$
c. $\sqrt{4x^2y}$	3. $2xy^2$
d. $\sqrt{16xy^2}$	4. $2x\sqrt{2y}$
e. $\sqrt{8xy^2}$	5. $4y\sqrt{x}$

3-7. Bonnie and Dylan were both working on simplifying the expression at right. Each of their first steps is shown below. Help (Html5)⇔Help (Java)

$$\left(\frac{2x^5y^4}{8xy^3}\right)^3$$

Bonnie: $\dfrac{8x^{15}y^{12}}{512x^3y^9}$ Dylan: $\left(\dfrac{x^4y}{4}\right)^3$

Each of them is convinced that they have started the problem correctly. Has either of them made an error? If so, explain the error completely. If not, explain how they can both be correct and verify that they will get the same, correct solution. Which student's method do you prefer? Why?

3-8. Describe the graphs of the equations given in parts (a) and (b) below. What are their domains and ranges? Help (Html5)⇔Help (Java)

a. $y = 3$

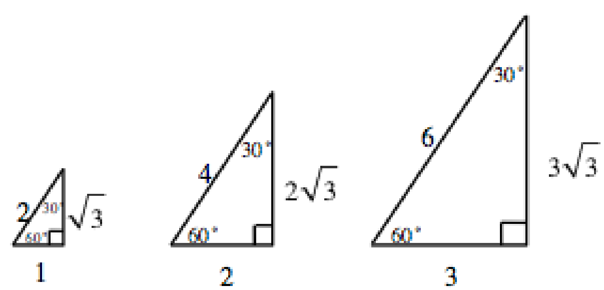
b. $x = -2$

c. Where do the two graphs cross?

3-9. Solve this system for m and b: Help (Html5)⇔Help (Java)

$$342 = 23m + b$$
$$147 = 10m + b$$

3-10. Tanika made this sequence of triangles: Help (Html5)⇔Help (Java)



a. If the pattern continues, what do you think the next two triangles in the sequence would be?

b. Write a sentence to explain how to find the long leg and hypotenuse if you know the short leg (i.e., if the base is n units long).

3-11. Consider the sequence 3, 9, … Help (Html5)⇔Help (Java)

a. Assuming that the sequence is arithmetic with $t(1)$ as the first term, find the next four terms of the sequence and then write an equation for $t(n)$.

b. Assuming that the sequence is geometric with $t(1)$ as the first term, find the next four terms of the sequence and then write an equation for $t(n)$.

c. Create a sequence that begins with 3 that is neither arithmetic nor geometric. For your sequence, write the next four terms and, if you can, write a rule for $t(n)$.

3-12. Simplify each expression without using a calculator. Help (Html5)⇔Help (Java)

a. $25^{-1/2}$

b. $(\frac{1}{27})^{-1/3}$

c. $9^{3/2}$

d. $16^{-3/4}$