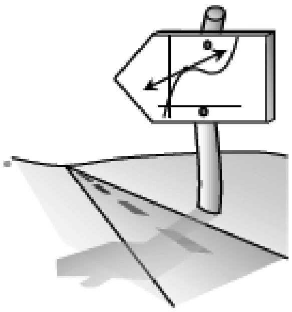
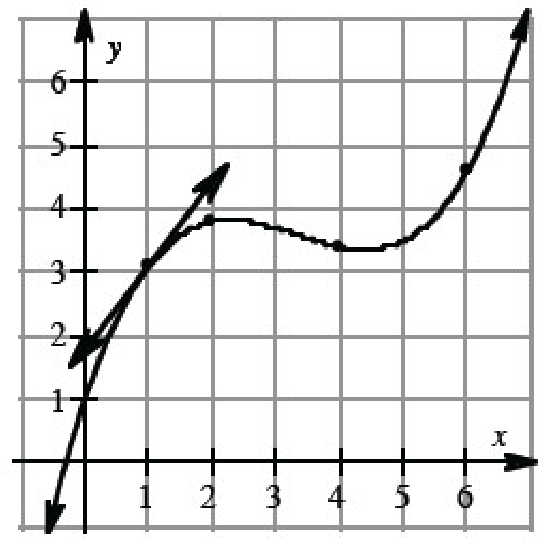
3.1.1 Who has the power?

The Power Rule



3-1. Notice that a tangent to the graph has been drawn at $x = 1$. On the Lesson 3.1.1 Resource Page provided by your teacher, carefully draw tangents to the curve at $x = 2$, $x = 4$, and $x = 6$ using a straightedge.



a. Write a slope statement for $f(x)$

b. Use the slope approximation technique you developed in the Ramp Lab in Lesson 2.3.1 to approximate the slope at $x = 2$.

c. If this graph represented the position of a roller coaster during its first 6 seconds, where was it moving the fastest? Where was it moving the slowest? How did you determine your answers?

d. How did the tangent lines help you answer the questions above?

3-2. On the resource page provided by your teacher, locate the graph of $f(x) = x^2$.

a. With a ruler, accurately draw a tangent to $f(x)$ for $x = -3, -2, -1, 0, 1, 2, 3$.

b. Using the same method you used in problem 3-1, find the slope of the tangent for each x-value and enter it into a table like the one below.

x	-3	-2	-1	0	1	2	3
m							

c. On the resource page, graph the data from the table in part (b) on the axes with the y-axis labeled $f'(x)$.

d. Use the table and the graph to find a **slope function,** $f'(x)$, a new function that gives the slope of the tangent to $f(x)$ for any x. What type of function is $f'(x)$?

3-3. On the resource page provided by your teacher, locate the graph of $f(x) = x^3$. Using a table of slope values, find and graph the **slope function,** $f'(x)$, a new function that gives the slope of the tangent to $f(x)$ for any x. You may want to do this part of the investigation on your graphing calculator by having to draw the tangent lines and calculate their slopes. What type of function is $f'(x)$?

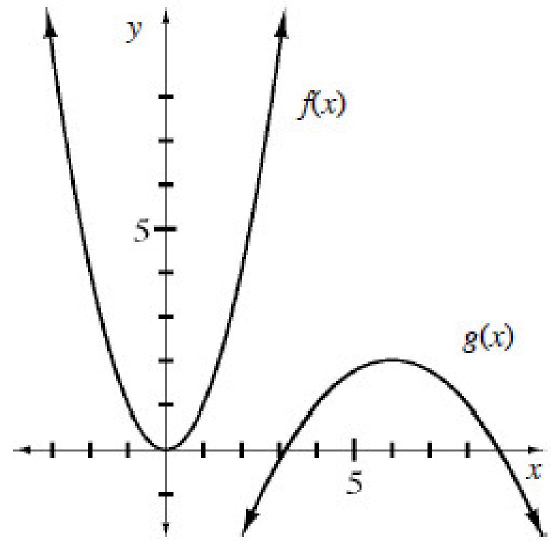
3-4. Similarly, find the slope function, $f'(x)$, for $f(x) = x$. Describe this slope function.

3-5. Recall what you know about the finite differences of cubic, quadratic, and linear functions. How does that compare to slope functions? Explain why.

3-6. THE POWER RULE: SLOPE FUNCTIONS FOR $f(x) = x^n$

Find a slope function $f'(x)$ for $f(x) = x^n$ when n is any positive integer. Show that your slope function works for more than one n-value. Add slope functions to your [Lesson 3.1.1 Resource Page] for $y = x$, $y = x^2$, and $y = x^3$.

3-7. SLOPE FUNCTIONS FOR $a(x - h)^n + k$



Now that you have a slope function for $f(x) = x^n$, we will find the slope function for $g(x) = a(x - h)^n + k$, when $f(x)$ is translated up, down, left, right, or stretched.

Try different transformations of $g(x)$ by selecting different values for a, h and k. When you are finished, you should be able to quickly find slope functions for parabolas, cubics, and other polynomials such as:

$g(x) = 3x^2$ $\quad g(x) = x^3 + 5$ $\quad g(x) = (x - 2)^6$

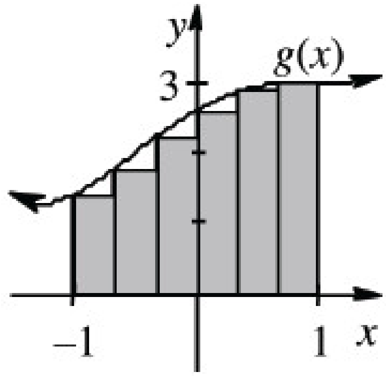
After testing your conjectures, state the slope function for the general polynomial $g(x) = a(x - h)^n + k$.

3-8. DERIVATIVE OF A SUM

What happens when we add polynomials? Write a conjecture regarding the slope function of $h(x) = f(x) + g(x)$. Then test your conjecture on $h(x) = 2x^{19} - x^3$. Alter your conjecture if necessary.

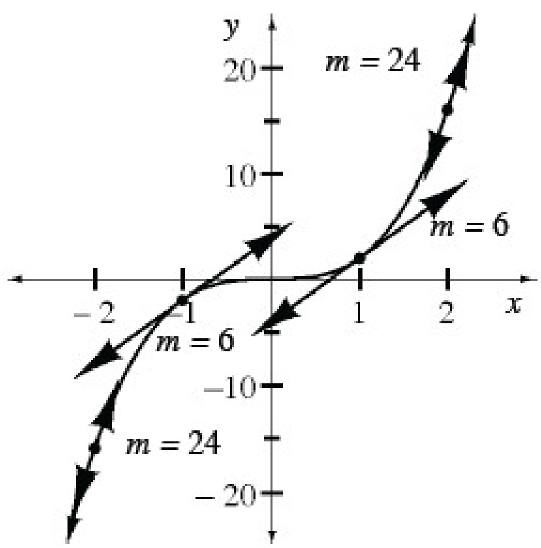
Review & Preview

3-9. Write an expression using sigma notation that represents the sum of the areas of the rectangles shown for the function below. Note, the rectangles have equal widths. Help (Html5)⇔Help (Java)



3-10. Create a *continuous* function that contains three pieces: one that is a sine curve, one that is a square root graph, and one that is a parabola. Write the function using correct notation. Help (Html5)⇔Help (Java)

3-11. Below is the graph of the function $f(x) = 2x^3$ with tangents drawn at $x = -2, -1, 1$, and 2. Use the slopes provided in the graph to find the slope function $f'(x)$. Notice that $f'(0) = 0$. It might be helpful to make a table of data relating x to m. Help (Html5)⇔Help (Java)

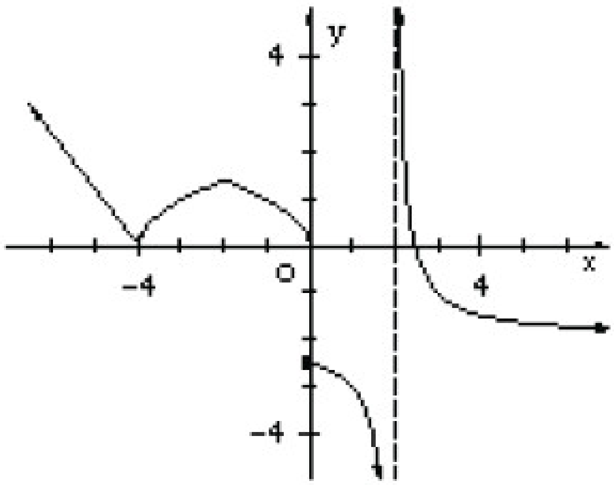


3-12. *Without your calculator*, find the limits indicated below. Help (Html5)⇔Help (Java)

a. $\lim\limits_{h \to 0} \dfrac{(2(x+h)-3)-(2x-3)}{h}$

b. $\lim\limits_{h \to 0} \dfrac{((x+h)^2+(x+h))-(x^2+x)}{h}$

3-13. Is the function graphed below continuous at the following values of x? If not, explain which conditions of continuity fail. Help (Html5)⇔Help (Java) x = −4, −2, 0, and 2



3-14. For the graph in problem 3-13, state the domain and range using interval notation. Help (Html5)⇔Help (Java)

3-15. Remember the conjecture developed in problem 3-5 and find the slope function, $f'(x)$, for each of the following functions. Help (Html5)⇔Help (Java)

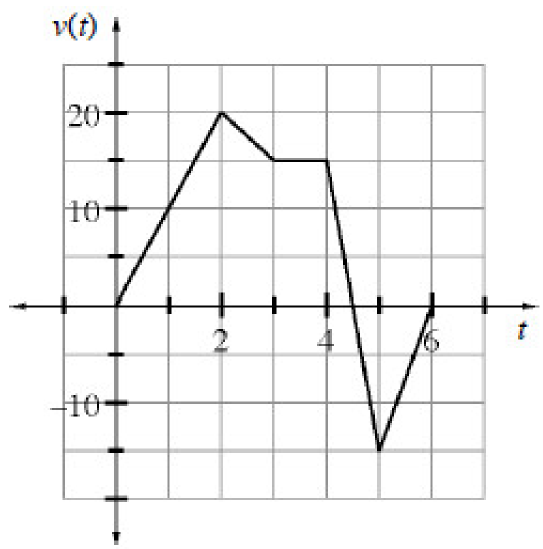
a. $f(x) = x^9$

b. $f(x) = x^{13}$

c. $f(x) = 2x$

d. $f(x) = 6$

3-16. After class, Stevie travels in a straight hallway with a velocity shown in the graph below right, where t is measured in minutes and v(t) is measured in feet per minute. Help (Html5)⇔Help (Java)



a. Explain what is happening when $t > 4.5$ minutes.

b. Calculate the total distance Stevie traveled.

c. If Stevie only travels in the straight hallway, how far does he end up from his original starting place?

d. What was Stevie's acceleration at $t = 1$?

e. When was Stevie's acceleration equal to zero?

3-17. Sketch $f(x) = \log|x|$. Help (Html5)⇔Help (Java)

a. Rewrite $f(x)$ as a piecewise function.

b. What is the domain of f?

3-18. Evaluate each limit. If the limit does not exist, say so but also state if y is approaching positive or negative infinity. Help (Html5)⇔Help (Java)

a. $\lim\limits_{x\to 2} \frac{x^2-x-2}{x^2-3x+2}$

b. $\lim\limits_{x\to 5} \frac{\sqrt{x}-\sqrt{5}}{x-5}$

c. $\lim\limits_{x\to 2^-} \frac{(2x+1)(x-5)^6}{(x-2)^7\sqrt{9-x}}$

d. $\lim\limits_{x\to \infty} \frac{4x^2+2x+9}{-(3x-6x^2+2)}$