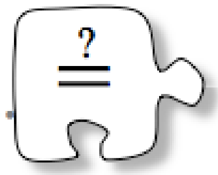
3.1.2 How can I rewrite it?



Rewriting Expressions and Determining Equivalence

In this lesson, you will continue to think about equivalent expressions. You will use an area model to demonstrate that two expressions are equivalent and to find new ways to write expressions. As you work with your team, use the following questions to help focus your discussion.

How can we be sure they are equivalent?

How would this look in a diagram?

Why is this representation convincing?

3-13. Jonah and Graham are working together. Jonah claims that $(x + y)^2 = x^2 + y^2$. Graham is sure Jonah is wrong, but he cannot figure out how to show it.

a. Help Graham find as many ways as possible to convince Jonah that he is incorrect. How can he rewrite $(x + y)^2$ correctly?

b. Are there any values for x and y for which $(x + y)^2 = x^2 + y^2$? In other words, is $(x + y)^2 = x^2 + y^2$ sometimes true? Justify your answer.

3-14. Do you think that an area model can help rewrite expressions that involve multiplication?

$+1$	$2x$	-3
$3x$	$6x^2$	$-9x$
	$2x$	-3

a. The area model at right relates the expressions $(2x - 3)(3x + 1)$ and $6x^2 - 7x - 3$. With your team, discuss how it can be used to show that these expressions are equivalent. Be prepared to explain your ideas.

b. Use an area model to find an expression equivalent to $(5k - 3)(2k - 1)$.

c. Use an area model to write a product that is equivalent to $x^2 - 3x - 4$.

3-15. Rewrite each of the following products as a sum and each sum as a product, drawing an area model when appropriate.

a. $2x^2 + 5x + 2$

b. $(3x - 1)(x + 2y - 4)$

c. $(x - 3)(x + 3)$

d. $4x^2 - 49$

e. $(p^2 + 3p + 9)(2p - 1)$

f. $(4 - x)(x^2 + 1) + (3x - 5)$

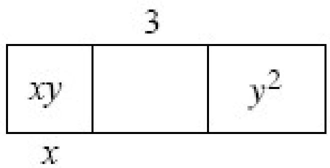
3-16. With your team, decide whether the following expressions can be represented with a model and rewrite each expression. Be prepared to share your strategies with the class.

a. $p(p + 3)(2p - 1)$

b. $x(x + 1) + (3x - 5)$

3-17. Copy each area model below and fill in the missing parts. Then write the two equivalent expressions represented by each model. Be prepared to share your reasoning with the class.

a.



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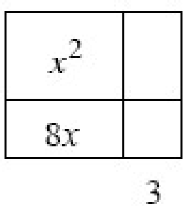

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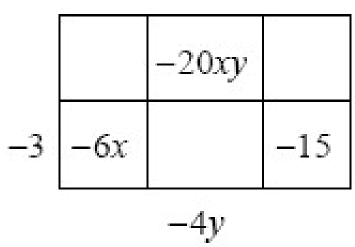
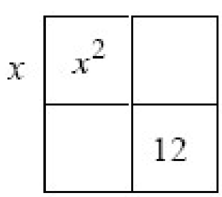


d.



3-18. Shinna noticed a similarity in parts (c) and (d) of problem 3-15.

a. Look back at those two problems and their rewritten form. What might Shinna have noticed? Discuss this with your team and be prepared to share your ideas with the class.

b. Shinna thinks she has found a shortcut that will allow her to rewrite expressions such as those written below without drawing a diagram. What do you think she has figured out? Try your ideas on the expressions shown below.

   i. $w^2 - 81$

   ii. $4m^2 - 1$

   iii. $x^2 - 16y^2$

3-19. Shinna has noticed that differences of squares can be factored easily.

a. Decide which of the expressions below can be seen as a difference of squares and can therefore be factored using Shinna's shortcut. For each difference of squares, show the squares clearly and then write the product. For example, $16x^2 - 9y^2$ can be rewritten as $(4x)^2 - (3y)^2$ and then as $(4x - 3y)(4x + 3y)$.

   i. $a^2 - 4b^2$

   ii. $2x^2 - 16$

   iii. $-x^2 + y^4$

   iv. $4a^2 + 9b^2$

b. Write two more expressions of your own that are differences of squares and show each in factored form.

3-20. Shinna wants to factor $9x^2y^4 - z^6$. "*Wait!*" she says. "*I think I can see a way to use my shortcut!*"

a. Discuss this with your team. Is Shinna's expression a difference of squares? If so, what are the squares? If not, explain why. Be ready to share your ideas with the class.

b. Shinna decided to rewrite her expression so that its structure was simpler to see. She wrote $9x^2y^4 - z^6$ as $U^2 - V^2$. What was she using U to represent? What about V?

c. George is confused! "*Shinna,*" he says, "*There was no U or V in your problem! What are you doing?*" Explain to George what is going on.

d. Help Shinna finish factoring the expression $9x^2y^4 - z^6$ by factoring $U^2 - V^2$ and then substituting the original expressions for U and V.

3-21. How can you use this method of substitution to make use of what you know about other expressions? Work with your team to describe the structure of each of the expressions in parts (a) through (d) below. Use substitution, when appropriate, to make the structure clear. For example, $25x^2 - 100y^4$ is a difference of squares and can be rewritten as $U^2 - V^2$ with $U = 5x$ and $V = 10y^2$.

The following questions might be useful:

What do all of these expressions have in common?

How might we substitute U and V to make rewriting simpler?

a. $a^2 + 2ab + b^2$

b. $x^2 - 6x + 9$

c. $9x^2 + 30xy + 25y^2$

d. $(a + 7)^2 - 10(a + 7) + 25$

3-22. Now it's your turn!

a. Work with a partner to write two really complicated-looking expressions that can actually be rewritten in a different form using substitution. Be sure to write the solutions for your expressions on a separate paper, so that you will be ready to trade expressions with another pair of students.

b. When you and your partner have been given another pair's expressions, use substitution to rewrite them. Do not let them stump you!

Review & Preview

3-23. Decide whether each of the following pairs of expressions are equivalent for all values of x (or a and b). If they are equivalent, show how you can be sure. If they are not, justify your reasoning completely. Help (Html5)⇔Help (Java)

a. $(x + 3)^2$ and $x^2 + 9$

b. $(x + 4)^2$ and $x^2 + 8x + 16$

c. $(x + 1)(2x - 3)$ and $2x^2 - x - 3$

d. $3(x - 4)^2 + 2$ and $3x^2 - 24x + 50$

e. $(x^3)^4$ and x^7

f. ab^2 and a^2b^2

3-24. Look back at the expressions in problem 3-23 that are not equivalent. For each pair of expressions, are there any values of the variable(s) that would make the two expressions equal? Justify your reasoning. Help (Html5)⇔Help (Java)

3-25. Jenna wants to solve the equation $2000x - 4000 = 8000$. Help (Html5)⇔Help (Java)

a. What easier equation could she solve instead that would give her the same solution? (In other words, what equivalent equation has easier numbers to work with?)

b. Justify that your equation in part (a) is equivalent to $2000x - 4000 = 8000$ by showing that they have the same solution.

c. Now Jenna wants to solve $\frac{3}{50} - \frac{x}{50} = \frac{7}{50}$. Write and solve an equivalent equation with easier numbers that would give her the same answer.

3-26. Find an equation for each sequence below. Then describe its graph. Help (Html5)⇔Help (Java)

a.

n	$t(n)$
3	8
5	2
7	−4

b.

n	$t(n)$
1	40
2	32
3	25.6

3-27. For the function $h(x) = -3x^2 - 11x + 4$, find the value of $h(x)$ for each value of x given below. Help (Html5)⇔Help (Java)

a. $h(0)$

b. $h(2)$

c. $h(-1)$

d. $h(\frac{1}{2})$

e. For what value(s) of x does $h(x) = 0$?

3-28. Find the x-intercepts for the graph of $y - x^2 = 6x$. Help (Html5)⇔Help (Java)

3-29. Multiply each pair of polynomial functions below to find an expression for $f(x) \cdot g(x)$. Help (Html5)⇔Help (Java)

a. $f(x) = 2x$, $g(x) = (x + 3)$

b. $f(x) = (x + 3)$, $g(x) = (x - 5)$

c. $f(x) = (2x + 1)$, $g(x) = (x - 3)$

d. $f(x) = (x + 3)$, $g(x) = (x + 3)$

3-30. Describe how the graph of $y + 3 = -2(x + 1)^2$ is different from $y = x^2$. Help (Html5)⇔Help (Java)

3-31. Given the parabola $f(x) = x^2 - 2x - 3$, complete parts (a) through (c) below. Help (Html5)⇔Help (Java)

a. Find the vertex by averaging the x-intercepts.

b. Find the vertex by completing the square.

c. Find the vertex of $f(x) = x^2 + 5x + 2$ using your method of choice.

d. What are the domain and range for $f(x) = x^2 + 5x + 2$?

3-32. Simplify each of the following expressions, leaving only positive exponents in your answer. Help (Html5)⇔Help (Java)

a. $(x^3y^{-2})^{-4}$

b. $-3x^2(6xy - 2x^3y^2z)$

3-33. Determine if each of the following functions are odd, even or neither. Help (Html5)⇔Help (Java)

a. $y = 3x^3$

b. $y = x^2 + 16$

c. $y = \frac{x^4}{2}$

3-34. You decide to park your car in a parking garage that charges \$3.00 for the first hour and \$1.00 for each hour (or any part of an hour) after that. Help (Html5)⇔Help (Java)

a. How much will it cost to park your car for 90 minutes?

b. How much will it cost to park your car for 118 minutes? 119 minutes?

c. How much will it cost to park your car for 120 minutes? 121 minutes?

d. Graph the cost in relation to the length of time your car is parked.

e. Is this function continuous?

f. Describe how the graph of this function will change if the parking garage raises their parking rate so that the first hour is now \$5.00.

3-35. Give the equation of each circle below in graphing form. Help (Html5)⇔Help (Java)

a. A circle with radius of 12 centered at the point (−2, 13).

b. A circle with center $(-1, -4)$ and radius 1.

c. A circle with equation $x^2 + y^2 - 6x + 16y + 57 = 0$. (Hint: Complete the square for both x and y.)

3-36. Giuseppe decides that he really wants some ice cream, so he leaves the house at 3:00 p.m. and walks to the ice cream parlor. He arrives at 3:15 (the ice cream parlor is 6 blocks away). He buys an ice cream cone and sits down to eat it. At 3:45 he heads back home, arriving at 4:05. Find Giuseppe's average walking rate in blocks per hour for each of the following situations. Help (Html5)⇔Help (Java)

a. His trip to the ice cream parlor.

b. His trip back home.

c. The entire trip including the time spent eating.