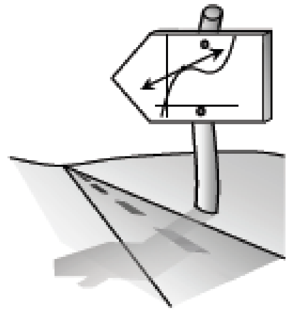
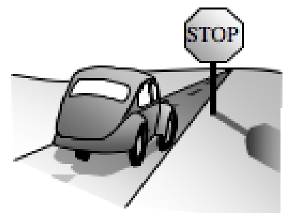
3.1.2 Why tangents?

Secants to Tangents, AROC to IROC



MATH NOTES



Power Rule

Since the beginning of this chapter, you have used the **Power Rule** to find the slope functions of polynomials of the form x^n when n is a positive integer. However, the Power Rule extends to all real values of n.

When $f(x) = x^n$, then $f'(x) = nx^{n-1}$ for all real values of n.

3-19. For which of the functions below can we apply the Power Rule from problem 3-5?

a. $y = \frac{1}{x}$

b. $y = x^5$

c. $y = \sqrt{x}$

d. $y = 2^x$

e. $y = 4x^0$

3-20. Use the results from problems 3-5, 3-7, and 3-8 to find $g'(x)$ for each function.

a. $g(x) = 2x^3$

b. $g(x) = x^8 - x^2$

c. $g(x) = -4x^3 - 2x + 5$

d. $g(x) = 6(x + 2)^4$

e. $g(x) = (x + 7)^{10} - 12x^5$

f. $g(x) = 2(x - 3)^3 + 4(x + 1)^2$

3-21. In Chapter 1, you discovered that for any function $f(x)$, the slope of a *secant* line at $x = a$, $\frac{f(x)-f(a)}{x-a}$, can be used to determine the **average rate of change** (AROC) over an interval. For example, average velocity, $\frac{\Delta\text{distance}}{\Delta\text{time}}$, represents average rate of change. Today you will investigate the meaning of the slope of a *tangent* line at a point of tangency on a curve. Before you begin, think back to your study of geometry. How is a tangent line different from a secant line?

3-22. A ladybug travels along a straight line at a distance of $10t^2$ millimeters in the first t seconds after it starts.

a. Sketch a graph of the bug's position as a function of time.

b. Sketch secant lines on the intervals below.

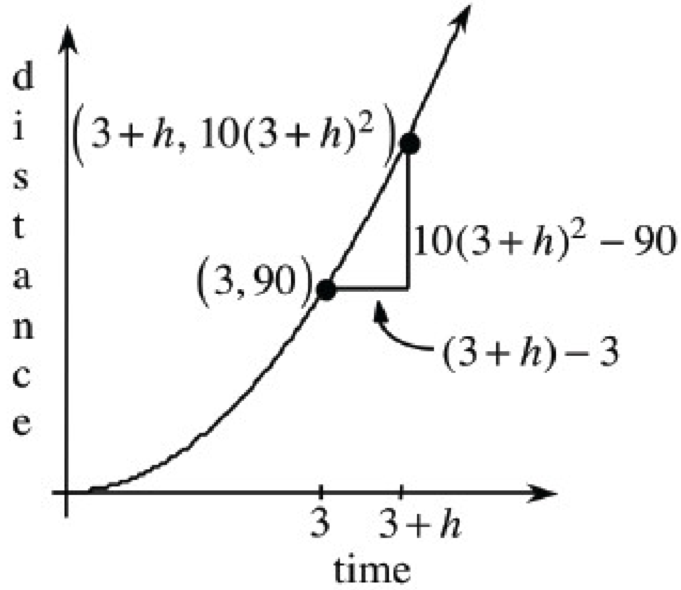
c. Label these secant lines with their slopes, i.e. average velocity / average rate of change (AROC).

i. [3,4]

ii. [3, 3.1]

iii. [3, 3.01]

3-23. We want to generalize the procedure you used in the last problem to any function over any interval. We will begin by computing the average velocity of our ladybug ($d(t) = 10t^2$) from $t = 3$ to $t = 3 + h$. As always, we compute the average velocity by dividing the change in distance (Δd) by the change in time (Δt).



a. What is the change in time over the specified interval?

b. Show that the change in distance over this time interval is $60h + 10h^2$.

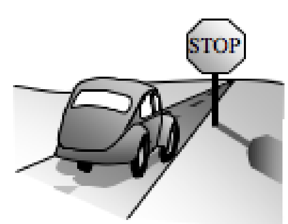
c. What is the average velocity over this time interval?

3-24. In the previous problem you approximated the ladybugs velocity at $t = 3$ by calculating the slope of the secant line between 3 and $3 + h$.

a. What would the line look like if h approached 0? With your team, make a prediction about the slope of this line. Justify your prediction.

b. Throughout this course, we have used the slope of the secant line to calculate the average rate of change, as in the Ramp Lab. What would the slope of the tangent line calculate?

MATH NOTES

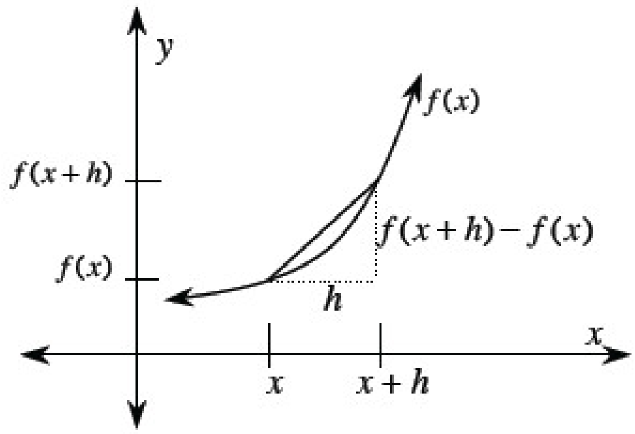


AROC (Average Rate of Change) and IROC (Instantaneous Rate of Change)

The **average rate of change** for $f(x)$ in the interval $[x, x+h]$ is:

$$\text{AROC} = \frac{f(x+h)-f(x)}{(x+h)-x} = \frac{f(x+h)-f(x)}{h}$$

The **instantaneous rate of change** for $f(x)$ in the interval $[x, x + h]$ is:



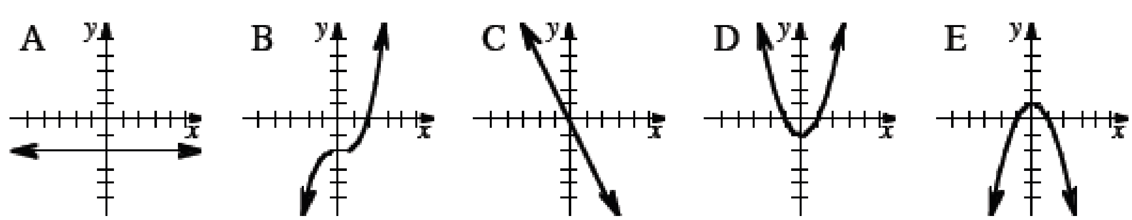
$$\text{IROC} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{(x+h)-x} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

3-25. Suppose $s(t)$ represents the distance a cockroach has traveled along a straight line up to time t. We know that the slope of the secant line of its graph represents the average velocity of the cockroach.

a. Describe how the *instantaneous* velocity (IROC) can be represented on distance graph?

b. Use the Math Note above to write an expression to describe the instantaneous velocity of the cockroach at t = 12 seconds.

3-26. Examine the graphs below. Some are slope functions of others. For each graph, determine whether one of the remaining graphs could be its slope function. If its slope function is not one of the options, sketch what it should look like.



A.

Review & Preview

3-27. If $f(15) = -3$ and $f(20) = 4$, *must* $f(x)$ have a root between $x = 15$ and 20? Explain why or why not. Be sure to include sketches that support your reasoning. Help (Html5)⇔Help (Java)

3-28. Find slope functions, $f'(x)$, for the following functions: Help (Html5)⇔Help (Java)

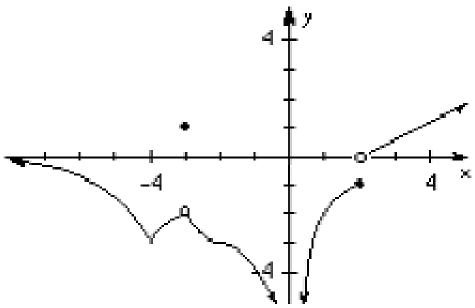
a. $f(x) = 7x^2$

b. $f(x) = \pi^2$ (Careful!)

c. $f(x) = 2(x - 2)^4 + 18x$

d. $f(x) = \frac{1}{3}x^6 + 2x^4 - 3$

3-29. Is the function graphed below continuous at the following values of x? If not, explain which conditions of continuity fail. Help (Html5)⇔Help (Java)
$x = -4, -3, 0$, and 2



3-30. For the graph in problem 3-29, state the domain and range using interval notation. Help (Html5)⇔Help (Java)

3-31. Evaluate the following limits. Help (Html5)⇔Help (Java)

a. $\lim_{x \to 4^-} (3x^2)$

b. $\lim_{x \to 3^+} (6 - 2x)$

c. $\lim_{x \to \infty} (\sqrt{x})$

d. $\lim_{x \to \infty} \left[\frac{1}{x^2}\right]$

3-32. Jasmin rolled a ball down a very steep ramp and got the distance function $s(t) = 2.3t^2$, where t is measured in seconds and $s(t)$ is measured in feet. Sketch a graph of her distance function on your paper. Then, carefully approximate the speed of the ball at $t = 3$ seconds. Help (Html5)⇔Help (Java)

3-33. Expand and evaluate the following sums. Help (Html5)⇔Help (Java)

a. $\sum_{i=2}^{6} (3i)$

b. $\sum_{i=1}^{10}\left(5+\frac{1}{2i}\right)$

3-34. In many textbooks the derivative (or IROC) is described in terms of x and Δx (delta x) instead of x and h.
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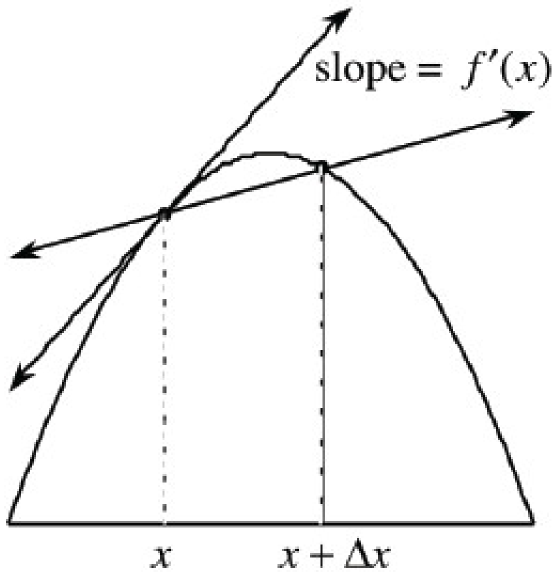

a. Explain why h and Δx are equivalent.

b. Use the picture above and the definition of the derivative to write the slope of the tangent line in terms of x and Δx.