

### 3.1.3 How can I solve it?

#### Solving by Rewriting



In the past few lessons, you have worked on recognizing and finding equivalent expressions. In this lesson, you will apply these ideas to solve equations. As you work, use the questions below to keep your team's discussion productive and focused.

How can we make it simpler?

Does anyone see another way?

How can we be sure the equations are equivalent?

**3-37.** Graciela was trying to solve the quadratic equation  $x^2 + 2.5x - 1.5 = 0$ . “*I think I need to use the Quadratic Formula because of the decimals,*” she told Walter. Walter replied, “*I’m sure there’s another way! Can’t we rewrite this equation so there aren’t any decimals?*”

- What is Walter talking about? Rewrite the equation so that it has no decimals.
- Rewrite your equation again, this time expressing it as a product.
- Now solve your new equation. Be sure to check your solution(s) using Graciela's original equation.

#### **3-38. SOLVING BY REWRITING**

Rewriting  $x^2 + 2.5x - 1.5 = 0$  in problem 3-37 gave you a new, equivalent equation that was much easier to solve. With your team, find an equivalent equation or system that you think might be easier to solve for parts (a) through (f) below. Then solve your new equation or system and check your answer(s) using the original equations.

- $100x^2 + 100x = 2000$
- $15x + 10y = -20$   
 $7x - 2y = 24$
- $\frac{1}{3}x^2 + \frac{x}{2} - \frac{1}{3} = 0$
- $\frac{4}{x^2} + \frac{12}{x} + 9 = 0$
- $\frac{x-3}{x} + \frac{2}{x-1} = \frac{5-x}{x}$

$$\begin{aligned} \text{f. } \frac{\sqrt{x^2-15x}}{2y} &= 5 \\ 3\sqrt{x^2-15x} - 3y &= 27 \end{aligned}$$

**3-39.** Graciela and Walter were working on solving the system of equations in part (f) of problem 3-38. They tried to rewrite both equations in  $y =$  form so that they could set them equal to each other.

$$\begin{aligned} 2y \cdot \frac{\sqrt{x^2-15x}}{2y} &= 2y \cdot 5 &\Rightarrow & \sqrt{x^2-15x} = 10y &\Rightarrow & y = \frac{\sqrt{x^2-15x}}{10} \\ \frac{3\sqrt{x^2-15x}-3y}{3} &= \frac{27}{3} &\Rightarrow & \sqrt{x^2-15x} - y = 9 &\Rightarrow & y = \sqrt{x^2-15x} - 9 \end{aligned}$$

Graciela and Walter realized they had a big mess to try to solve. “Wait,” Graciela said. “There’s an easier way. Let’s use substitution to make this system simpler!”

- Discuss this idea with your team. Does it make sense?
- Walter and Graciela decided to try this new idea, but they were not sure the best choice for what expression to replace with a new variable. They came up with these two options:

$$U = x^2 - 15x \quad \text{or} \quad U = \sqrt{x^2 - 15x}$$



To help Graciela and Walter decide, rewrite the original system from problem 3-38 part (f) twice, each time using a different version of  $U$ . Which version of  $U$  looks like it will make the system easier to solve?

- Solve your new system for  $U$  and  $y$ .
- Now what? Since your job in solving a system in  $x$  and  $y$  is to find values for both of those variables, you are not done. Work with your team to find a way to get the value of  $x$  from the value you found for  $U$ . Be ready to share your strategies with the class.

**3-40.** Consider each of the following equations and systems. Would substitution make them easier to solve? What expression might you temporarily replace with  $U$ ? Be ready to share your ideas on substitution with the class. You do not need to actually solve the equation(s).

$$\text{a. } (m^2 + 5m - 24)^2 - (m^2 + 5m - 24) = 6$$

$$\begin{aligned} \text{b. } 2x + y^7 &= 6 \\ 3x - 2y^7 &= -5 \end{aligned}$$

$$\text{c. } (4x^2 + 4x - 3)^2 = (x^2 - 5x - 6)^2$$

### 3-41. MORE EQUIVALENT EQUATIONS

Rewrite each of the following equations in another form by solving for  $y$ . (That is, rewrite the equations in  $y =$  form.) Check to be sure your new equation is equivalent to the original equation.

a.  $5x - 2y = 8$

b.  $xy + 3x = 2$

**3-42.** Rewrite the equation from part (b) of problem 3-41 in yet another form by solving for  $x$ . Be ready to share your strategies with the class.

**3-43.** None of the three equations below are equivalent. Show that this is true by rewriting the equations with an equivalent equation.

$$2x = 2y - 6$$

$$xy + 2x = (y + 2)(y + 3)$$

$$-x = -y - 3$$

**3-44.** Angelica and D'Lee were working on finding roots of two quadratic equations:

$$y = (x - 3)(x - 5) \text{ and } y = 2(x - 3)(x - 5).$$

Angelica made an interesting claim: “Look,” she said, “When I solve each of them for  $y = 0$ , I get the same solutions. So these equations must be equivalent!”



D'Lee is not so sure. “How can they be equivalent if one of the equations has a factor of 2 that the other equation doesn't?” she asked.

- Who is correct? Is  $y = (x - 3)(x - 5)$  equivalent to  $y = 2(x - 3)(x - 5)$ ? How can you justify your ideas using tables and graphs?
- Are the solutions of  $0 = (x - 3)(x - 5)$  equivalent to the solutions of  $0 = 2(x - 3)(x - 5)$ ? Again, how can you justify your ideas?



## METHODS AND MEANINGS

### MATH NOTES

## Vocabulary for Expressions

A mathematical expression is a combination of numbers, variables, and operation symbols. Addition and subtraction separate expressions into parts called terms. For example,  $4x^2 - 3x + 6$  is an expression. It has three terms:  $4x^2$ ,  $3x$ , and 6. The **coefficients** of the terms with variables are 4 and  $-3$ . 6 is called a **constant term**.

A single-variable **polynomial** is an expression that involves, at most, the operations of addition, subtraction, and multiplication. Most of the polynomials you will work with can be written as expressions with terms of the following form:

$$(\text{any real number})x^{(\text{whole number})}$$

For example,  $4x^2 - 3x^1 + 6x^0$  is a polynomial, as is the simplified form,  $4x^2 - 3x + 6$ . Also, since  $6x^0 = 6$ , 6 itself is a polynomial.

The function  $f(x) = 7x^5 + 2.5x^3 - \frac{1}{2}x + 7$  is a polynomial function.

A **binomial** is a polynomial with only two terms, for example,  $x^3 - 0.5x$  and  $2x + 5$ .

The following expressions are *not* polynomials:  $2^x - 3$ ,  $\frac{1}{x^2 - 2}$ , and  $\sqrt{x - 2}$ .

An expression that can be written as the quotient of two polynomials is a **rational expression**. For example,  $\frac{1}{x^2 - 2}$  is a rational expression.



**3-45.** Rewrite each equation below. Then solve your new equation. Be sure to check your solution using the original equation. [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

a.  $(n + 4) + n(n + 2) + n = 0$

b.  $\frac{4}{x} = x + 3$

**3-46.** Decide whether each of the following pairs of expressions or equations are equivalent. If they are, show how you can be sure. If they are not, justify your reasoning completely. [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

a.  $(ab)^2$  and  $a^2b^2$

b.  $3x - 4y = 12$  and  $y = \frac{3}{4}x - 3$

c.  $y = 2(x - 1) + 3$  and  $y = 2x + 1$

d.  $(a + b)^2$  and  $a^2 + b^2$

e.  $\frac{x^6}{x^2}$  and  $x^3$

f.  $y = 3(x - 5) + 2$  and  $y = 2x - 8$

**3-47.** Look back at the expressions in problem 3-46 that are not equivalent. Are there any values of the variables that would make them equal? Justify your reasoning. [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

**3-48.** Find the formula for  $t(n)$  for the arithmetic sequence in which  $t(15) = 10$  and  $t(63) = 106$ . [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

**3-49.** Jillian's parents bought a house for \$450,000, and the value of the house has been increasing steadily by 3% each year. [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

- Find the formula  $t(n)$  that represents the value of the house each year.
- If Jillian's parents sell their house 10 years after they bought it, how much profit will they make? (That is, how much more are they selling it for than they bought it for?) Express your answer as both a dollar amount and a percent of the original purchase price.

**3-50.** Factor  $5x^3y + 35x^2y + 50xy$  completely. Show every step and explain what you did. [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

**3-51.** While Jenna was solving the equation  $150x + 300 = 600$ , she wondered if she could first change the equation to  $x + 2 = 4$ . What do you think? [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

- Solve both equations and verify that they have the same solution.
- What did Jenna do to the equation  $150x + 300 = 600$  to change it to  $x + 2 = 4$ ?
- Use the same method to rewrite and solve  $60t - 120 = 300$ .

**3-52.** Consider the sequence 10, 2, ... [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

- Assuming that the sequence is arithmetic with  $t(1)$  as the first term, write the next four terms of the sequence and then write an equation for  $t(n)$ .
- Assuming that the sequence is geometric with  $t(1)$  as the first term, write the next four terms of the sequence and then write an equation for  $t(n)$ .
- Create a totally different sequence that begins 10, 2, ... For your sequence, write the next four terms and an equation for  $t(n)$ .

**3-53.** Rewrite each radical below as an equivalent expression using fractional exponents. [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

a.  $\sqrt[3]{5}$

b.  $\sqrt[3]{9}$

c.  $\sqrt[8]{17^x}$

d.  $7\sqrt[4]{x^3}$

**3-54.** Give the equation of each circle below in graphing form. [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

- a. A circle with center  $(0, 0)$  and radius 6.
- b. A circle with center  $(2, -3)$  and radius 6.
- c. A circle with equation  $x^2 + y^2 - 8x + 10y + 5 = 0$ .

**3-55.** If the cooling system in a light-water nuclear reactor is shut off, the temperature of the fuel rods will increase. The temperature of the fuel rods during the first hour could be modeled by the equation  $T = 680(1.0004)^t - 655$ , where  $t$  is the time in seconds, and  $T$  is the temperature of the fuel rods in degrees Fahrenheit. Average rate of change can be calculated by finding the slope between two points. Find the average rate at which the temperature changes for the first 30 minutes. [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

**3-56.** In the year 2006, the average cost to rent a car was \$39 for the first day and an additional \$23 for each additional day. [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)



- a. Graph the relationship between cost and the duration of a car rental in 2006.
- b. Describe how the graph would be transformed if the current average cost of a car rental has increased to \$50 for the first day.