3.2.1 Where does the graph go?

Investigating Rational Functions

In your experience with algebra, you have added, subtracted, and multiplied polynomials, but what happens when you divide them? Today you will make some predictions about the graphs that result when two linear functions are combined by adding, subtracting, multiplying, or dividing them. You will further explore division of polynomials in Chapter 8.

3-57. COMBINING LINEAR FUNCTIONS INVESTIGATION

Your team will be assigned a pair of linear functions from the list below. 3-57 Student eTool (html5)

$f_1(x) = x - 2$	$f_2(x) = x + 3$	$f_3(x) = x - 4$
$g_1(x) = 2x + 3$	$g_2(x) = 5x - 9$	$g_3(x) = 5x + 8$
$f_4(x) = x - 1$	$f_5(x) = x - 3$	$f_6(x) = x + 4$
$g_4(x) = 2x + 5$	$g_5(x) = 5x - 9$	$g_6(x) = 5x + 6$
$f_7(x) = x - 3$	$f_8(x) = x + 3$	$f_9(x) = x + 2$
$g_7(x) = 2x + 5$	$g_8(x) = 5x + 7$	$g_9(x) = 5x + 3$

Your Task: With your team, find out as much as you can about what happens when you combine the two linear functions using each of the operations of addition, subtraction, multiplication, and division. Use the following steps to guide your investigation.

1. Make your own prediction of the shape of each new graph and draw a quick, rough sketch on your paper.
2. Discuss your prediction with your teammates.
3. Use a graphing calculator to check your team's prediction.
4. Summarize your findings.

Be sure to carefully record all of your work and be prepared to share your summary statements about the results for each operation with the class.

Note: When entering operations into the graphing calculator, you may need to insert extra parentheses so the calculator will follow your intended Order of Operations.

Discussion Points

What does the new graph look like?

What happens when we use the expressions in a different order? Why?

What are the domains and ranges of the new graphs?

Further Guidance

3-58. First investigate the graphs for the sum and difference of your two functions. Does the order of subtraction matter? What do you think would happen if you added or subtracted any two linear functions $f(x)$ and $g(x)$. Can you think of any exceptions?

3-59. Now multiply the functions and graph the product $f(x) \cdot g(x)$. How well did your team predict the result? What do you think the resulting graph would look like if you multiplied any two linear functions?

3-60. Divide the two functions, TRACE, and graph the result. Check the table of values or use the $\frac{f(x)}{g(x)}$ function on your calculator to find the x-values that are "holes" in your graph. (Your teacher will check if the graph in the standard window of your graphing calculator has extra lines that obscure the picture. If that is the case, the teacher will make some suggestions.)

Further Guidance section ends here.

3-61. CLOSED SETS

Whole numbers (positive integers and zero) are said to be a closed set under addition: if you add two whole numbers, you always get a whole number. Whole numbers are not a closed set under subtraction: if you subtract two whole numbers, you do not always get a whole number. For example, $2 - 5 = -3$ and -3 is not a whole number.

a. Investigate with your team whether the set of integers is a closed set under addition and under subtraction. Then investigate whether the integers are a closed set under multiplication and under division. Give examples. If you think the set is closed, explain why. If, not, give counterexamples.

b. Are single-variable polynomials closed under addition, subtraction, and multiplication? In other words, if you add, subtract, or multiply two polynomials that have the same variable, will you always get a polynomial as your answer? If you think the set is closed, explain why. If, not, give counterexamples.

3-62. With your team describe the graphs of $p(x)$ and $q(x)$ and then make an estimate of their sum, difference, product, and quotient functions before using a graphing tool.

$$p(x) = x^3 - 3x - 1 \qquad\qquad q(x) = x - 1$$

a. $p(x) + q(x)$

b. $p(x) - q(x)$

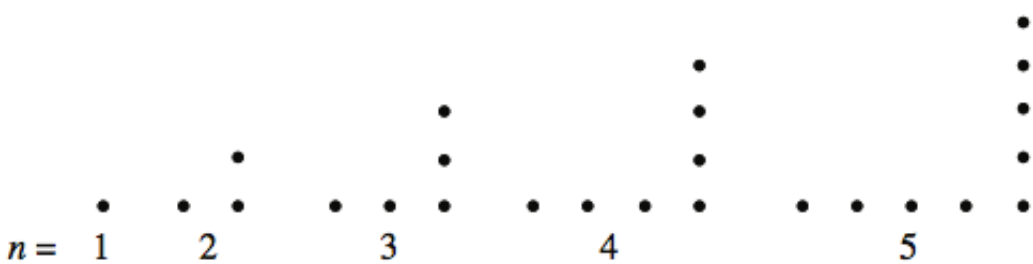
c. $p(x) \cdot q(x)$

d. $\frac{p(x)}{q(x)}$

e. For parts (a) through (d) above, sketch a careful graph and make a note of the shape, the domain and range, and any other characteristics you notice. Your team should be prepared to share their observations with the class.

Review & Preview

3-63. Given that n is the length of the bottom edge of the backward L-shaped figures below, what sequence is generated by the total number of dots in each figure? What is the 46th term, or, $t(46)$ of this sequence? The nth term? Help (Html5)⇔Help (Java)



3-64. A piece of metal at 20°C is warmed at a steady rate of 2 degrees per minute. At the same time, another piece of metal at 240°C is cooled at a steady rate of 3 degrees per minute. After how many minutes is the temperature of each piece of metal the same? Explain how you found your answer. Help (Html5)⇔Help (Java)

3-65. The price of a movie ticket averages \$10.25 and is increasing by 3% per year. Use that information to complete parts (a) through (c) below. Help (Html5)⇔Help (Java)

a. What is the multiplier in this situation?

b. Write a function that represents the cost of a movie ticket n years from now.

c. If tickets continue to increase at the same rate, what will they cost 10 years from now?

3-66. Use the meaning of an exponent to rewrite the expression $(y - 2)^3$. Help (Html5)⇔Help (Java)

3-67. This problem is a checkpoint for rewriting and simplifying expressions with integral and rational exponents. It will be referred to as Checkpoint 3A.

For parts (a) through (d), rewrite each expression. For parts (e) through (h), simplify each expression.

a. $\sqrt[5]{x}$

b. $\frac{1}{x^3}$

c. $x^{2/3}$

d. $\frac{1}{\sqrt{x}}$

e. $x^{-1}y^{-8}$

f. $(m^2)^{-3/2}$

g. $(x^3y^6)^{1/2}$

h. $(9x^3y^6)^{-2}$

Check your answers by referring to the Checkpoint 3A materials.

If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 3A materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do problems like these quickly and easily.

3-68. While David was solving the equation $100x + 300 = 500$, he wondered if he could first change the equation to $x + 3 = 5$. What do you think? Help (Html5)⇔Help (Java)

a. Solve both equations and verify that they have the same solution.

b. What could you do to the equation $100x + 300 = 500$ to change it into $x + 3 = 5$?

3-69. Multiply the expressions below using generic rectangles. Help (Html5)⇔Help (Java)

a. $(5m - 1)(m + 2)$

b. $(6 - x)(2 + x)$

c. $(5x - y)^2$

d. $3x(2x - 5y + 4)$