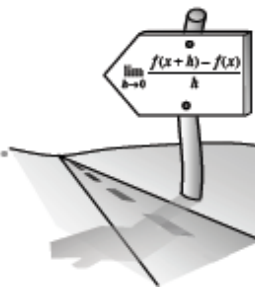


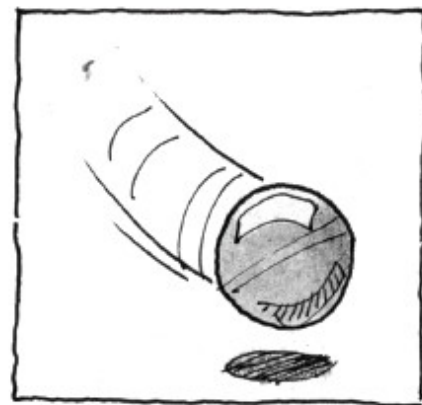
3.2.1 What is a derivative?

Definition of a Derivative



3-35. How does a graphing calculator find the slope of a tangent? Or, how did mathematicians find slopes before technology was available? In this lesson, we will examine slopes of secants and tangents by examining three different methods to find the exact slope of a tangency.

To start, revisit the use of secants by studying the different slopes when $x = 4$ in the Ramp Lab. Later, we will use secants to help us find slopes of tangents with precision.



Each of the following students used a different method to estimate the velocity of the ball at $t = 4$ seconds.

x (seconds)	1	2	3	4	5	6
$f(x)$ (meters)	0.2	0.8	1.8	3.2	5.0	7.2

Hana estimated $m \approx \frac{f(5) - f(4)}{5 - 4} = \frac{5.0 - 3.2}{5 - 4} = 1.8$.

Anah estimated the slope as $m \approx \frac{f(4) - f(3)}{4 - 3} = \frac{3.2 - 1.8}{4 - 3} = 1.4$.

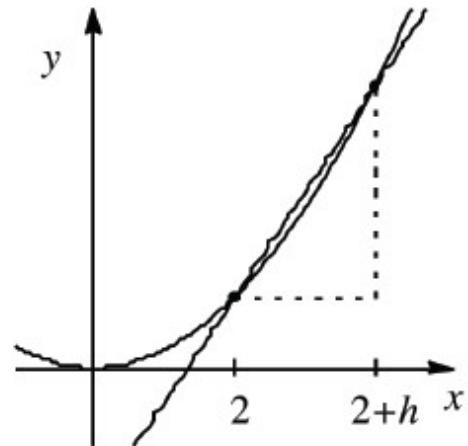
Hanah estimated the slope as $m \approx \frac{f(5) - f(3)}{5 - 3} = \frac{5.0 - 1.8}{5 - 3} = 1.6$.

- What are the units of the slopes found by each student? Explain why.
- Compare and contrast each method.
- Trace the graph below of $f(x)$ on your paper and sketch the secant lines that Hana, Anah and Hanah used to approximate the slope at $t = 4$.
- Is Hanah's slope the average of Hana and Anah's? Does this mean that Hana's method gives the best

approximation?

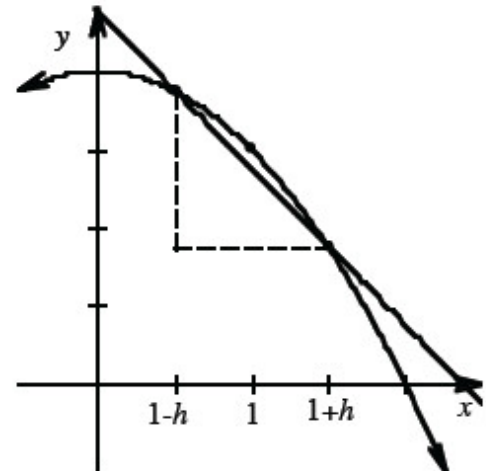
3-36. To approximate the slope of the curve $f(x) = 0.25x^2$ at $x = 2$, Devin picked another point on the curve, a small distance of h to the right of 2.

- If the second point is located at the variable point $(2 + h, f(2 + h))$, find and simplify an expression that represents the slope of this secant.
- Whose method did Devin use from problem 3-35: Hana's, Anah's or Hanah's?
- To get the exact slope of the tangent at $x = 2$, what should we do with h ? Write an expression that represents the true slope at $x = 2$.
- Use your expression from part (c) to find the slope of the curve at $x = 2$.
- Does it matter if h is positive? Why or why not?



3-37. Hanah wants to use her method to find the slope $f'(1)$ for $f(x) = 4 - x^2$. Hanah picks two points equidistant from 1 but very close to 1.

- Write and simplify an expression to represent the slope of Hanah's secant and then find the slope of the tangent.
- Use Hanah's method for $g(x) = x^2 - 11x + 2$, to find the slope of the tangent when $x = 3$.
- Find a slope function that represents the slope of $g(x)$ for all values of x .



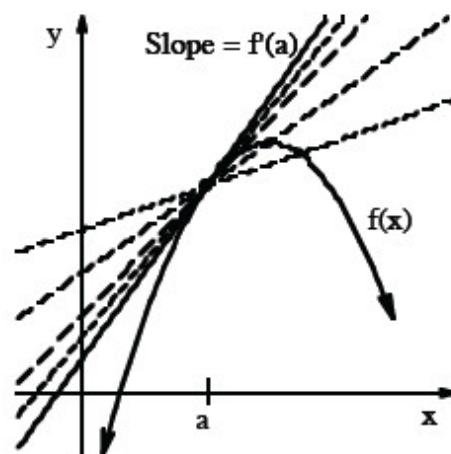
3-38. Use each method from problem 3-35 to find a generic algebraic expression which approximates the slope of a function $f(x)$ at any point x using increments of size h , where $h > 0$. For each expression, provide a sketch of a generic function $f(x)$ and the secant being represented.

MATH NOTES



The Derivative

A function that represents the slope of a function $f(x)$ at each x -value is called a **derivative function (or slope function)**.



The slope of a tangent to at any point x is called the **derivative** of $f(x)$ at x . It is found by finding a limit of the slope of a secant as $h \rightarrow 0$. The standard form of this type of limit is:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

If the slope (or instantaneous rate of change) at a particular x -value is desired, such as at $x = a$, then the notation used is $f'(a)$. This slope can be found by replacing x with a .

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

3-39. The Math Note above features the definition of the derivative using Hana's method. Write the definition of the derivative using Anah's method and the definition of the derivative at a point ($x = a$) using Hanah's method.

3-40. Use the definition of the derivative to find the slope function, $f'(x)$ of $f(x) = 4x^2 - 3$. Then use your slope function to find $f'(11)$ and $f'(1000)$.

3-41. Lulu used the limit below to find the derivative of some function $f(x)$:

$$f'(x) = \lim_{h \rightarrow 0} \frac{2(x+h)^3 - 2x^3}{h}$$

- What is $f(x)$?
- What is $f'(x)$?



3-42. Rewrite each of the following terms in the form of x^n . [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

- \sqrt{x}

b. $\frac{1}{x^3}$

c. $\sqrt{\frac{1}{x}}$

d. $x \cdot \sqrt[3]{x^2}$

3-43. Write and evaluate a Riemann sum to determine the area $A(f, -1 \leq x \leq 1)$ for $f(x)$ below. Choose the number of rectangles so that your answer will be a good approximation of the area. What is interesting about the sign of your result? Use a description or sketch of the function to support your answer. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

$$f(x) = x^4 - x^2$$

3-44. Given the function $\frac{x^2+2x-8}{x^2-2x}$, find the following limits *without using your graphing calculator*. [Help](#)
(Html5) \Leftrightarrow [Help](#) (Java)

a. $\lim_{x \rightarrow 6} f(x)$

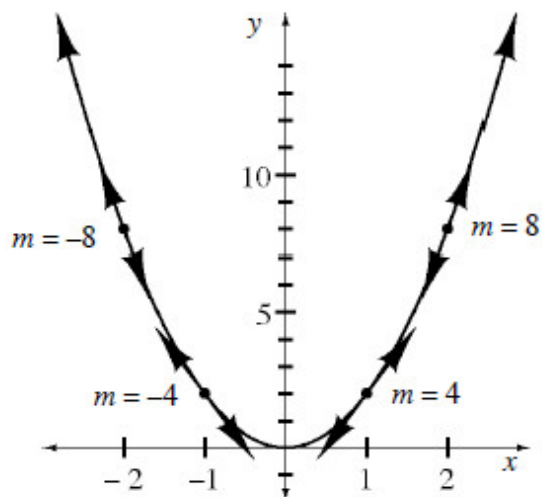
b. $\lim_{x \rightarrow 2} f(x)$

c. $\lim_{x \rightarrow \infty} f(x)$

d. $\lim_{x \rightarrow 0} f(x)$



3-45. Below is the graph of a function $f(x)$ with tangents drawn at $x = -2, -1, 1$, and 2 . $f'(0) = 0$. Use the slopes provided in the graph to find the slope function $f'(x)$. Also, find $f(x)$. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)



3-46. Find $A(h, 0 \leq x \leq 8)$ if $h(x) = -|x - 4| + 4$. Sketch the graph and shade the region. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

3-47. After completing the Ramp Lab in Chapter 2, Marquita decided to ride her bicycle down a nearby hill to gain more data on rolling objects. The data she collected is shown in the table below. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

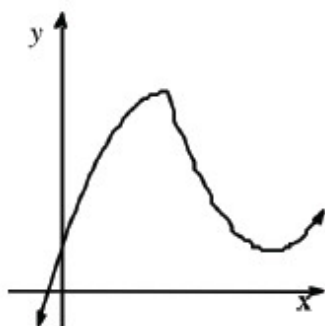
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Time (sec)	0.0	2.0	3.1	6.0	7.5	9.0
Distance (m)	0.0	3.8	9.1	32.7	49.7	69.5

- Can Marquita use this table to determine her exact velocity at $t = 6$. Explain.
- Approximate Marquita's velocity at $t = 6$ using Hana's method.
- Marquita's teacher observed that her data fits the function $d(t) = 0.789t^2 + 0.703t - 0.338$. Assuming that her teacher is correct, compute Marquita's *exact* velocity at $t = 6$ and $t = 10$.



3-48. Write a slope statement to describe the graph below. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)



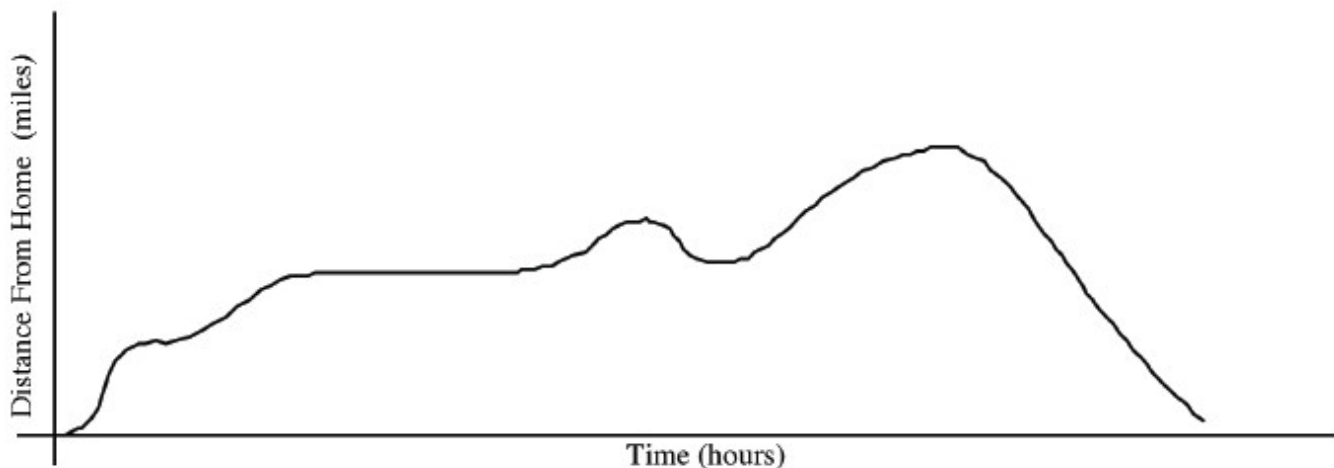
3-49. Find a tangent line that is a linear approximation to the function $y = \tan x$ at $x = \frac{\pi}{4}$. Using your graphing calculator, investigate over what x -values this approximation is reasonable. Record your observations. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

3-50. WHAT A DAY!

Below is a graph of the distance David traveled away from his home on a trip to the mountains. Place the events listed below in the proper order based on details from the graph.

Trace the graph below and identify the parts that correspond to each event during David's trip.

Then answer parts (a) through (d) below. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)



EVENTS

- David had to drive back to pick up his credit card that he forgot at the restaurant.
- David's car breaks down and he is towed back to a repair shop near his house.
- David stopped for gas and got a quick bite to eat.
- David got pulled over and received a speeding ticket. He then continued his trip at a slower rate.

- a. What are the units for the slope of the curve?
- b. What is significant about the slope of the curve when David is stopped?
- c. How does the slope of the curve tell you when David is speeding?
- d. Interpret the graph where the slope is negative. What is David doing then?

3-51. Evaluate each limit. If the limit does not exist, say so but also state if y is approaching positive or negative infinity.

[Help \(Html5\)](#) \leftrightarrow [Help \(Java\)](#)

- a. $\lim_{x \rightarrow -3^+} \frac{x-3}{x+3}$
- b. $\lim_{x \rightarrow 2} \frac{(x+2)^2 - 2}{x}$
- c. $\lim_{x \rightarrow \infty} \frac{2+2^x}{2-2^x}$
- d. $\lim_{x \rightarrow \pi^+} \pi$ (Careful!)