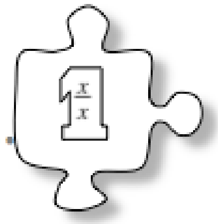
3.2.2 How can “1” be useful?

Simplifying Rational Expressions



In this chapter, you will focus on an important number: the number 1. What is special about 1? What can you do with the number 1 that you cannot do with any other number? You will use your understanding of the number 1 to simplify algebraic fractions, which are also known as **rational expressions.**

3-70. What do you know about the number 1? With your team, brainstorm and be ready to report your ideas to the class. Create examples to help show what you mean.

3-71. Mr. Wonder claims that anything divided by itself equals 1 (as long as you do not divide by zero).

a. Mr. Wonder states that $\frac{16x}{16x} = 1$ if x is not zero. What is his hypothesis and his conclusion?

b. Is Mr. Wonder correct? That is, is his statement true? Justify your conclusion.

c. Why can't x be zero?

d. Next he considers $\frac{x-3}{x-3}$. Does this equal 1? What value of x must be excluded in this fraction?

e. Create your own rational expression (algebraic fraction) that equals 1.

f. Mr. Wonder also says that when you multiply any number by 1, the number stays the same. For example, he says that the product below equals $\frac{x}{y}$. Is he correct?

$$\frac{z}{z} \cdot \frac{x}{y} = \frac{x}{y}$$

3-72. Use a calculator to graph the function $f(x) = \frac{16x}{16x}$. Use the trace button to trace along the line and notice what happens at $x = 0$. Is the expression $\frac{16x}{16x}$ equivalent to 1? Explain. Desmos GC

3-73. With your team, compare and contrast the graphs of each of the following functions: 3-73 Student eTool

$$f_1(x)=\frac{2x-3}{2x-3} \qquad f_2(x)=\frac{2x-3}{3-2x}$$

$$f_3(x)=\frac{2x-3}{2x+3} \qquad f_4(x)=\frac{1}{2x-3}$$

a. First visualize and make a quick sketch of what you imagine the graph of each will look like.

b. Discuss your sketches with the rest of your team.

c. Use calculators to graph each rational function, and adjust your sketches if needed.

d. Use the $\boxed{\text{TRACE}}$ function or the table on your graphing calculator to find the location of the "hole" in each of the graphs, and describe their similarities and differences. Include their domains and ranges in the descriptions.

3-74. Use what you know about the number 1 to simplify each expression below, if possible. State any value(s) of the variable that would make the denominator zero.

a. $\frac{x^2}{x^2}$

b. $\frac{x}{x}\cdot\frac{x}{x}\cdot\frac{x}{3}$

c. $\frac{x-2}{x-2}\cdot\frac{x+5}{x-1}$

d. $\frac{9}{x}\cdot\frac{x}{9}$

e. $\frac{h\cdot h\cdot k}{h}$

f. $\frac{(2m-5)(m+6)}{(m+6)(3m+1)}$

g. $\frac{6(n-2)^2}{3(n-2)}$

h. $\frac{3-2x}{(4x-1)(3-2x)}$

3-75. Mr. Wonder now tries to simplify $\frac{4x}{x}$ and $\frac{4+x}{x}$. Desmos GC

a. Mr. Wonder thinks that since $\frac{x}{x}$ = 1, then $\frac{4x}{x}$ = 4. Is he correct? Substitute three values of x to justify your answer.

b. He also wonders if $\frac{4+x}{x}$ = 5. Is this simplification correct? Substitute three values of x tor use your calculator to compare the graphs of $g(x) = \frac{4+x}{x}$ with $h(x) = 5$ to justify your answer. Remember that $\frac{4+x}{x}$ is the same as $(4 + x) \div x$.

c. Compare the results of parts (a) and (b). When can a rational expression be simplified in this manner?

d. Which of the following expressions below is simplified correctly? Explain how you know.

i. $\frac{x^2+x+3}{x+3} = x^2$

ii. $\frac{(x+2)(x+3)}{x+3} = x + 2$

3-76. In problem 3-75, you may have noticed that both the numerator and denominator of an algebraic fraction must be written as a product before you can use any of the terms to create a **Giant One** (a form of the number 1). Examine the expressions below. Factor the numerator and denominator of each fraction, if necessary. That is, rewrite each one as a product. Then look for "Giant Ones" and simplify. For each expression, assume the denominator is not zero.

a. $\frac{x^2+6x+9}{x^2-9}$

b. $\frac{2x^2-x-10}{3x^2+7x+2}$

c. $\frac{28x^2-x-15}{28x^2-x-15}$

d. $\frac{x^2+4x}{2x+8}$

3-77. LEARNING LOG

In your Learning Log, explain how to simplify rational expressions such as those in problem 3-76. Be sure to include an example. Title this entry "Simplifying Rational Expressions" and include today's date.

Review & Preview

3-78. Simplify the expressions below. Help (Html5)⇔Help (Java)

a. $\frac{x^2-8x+16}{3x^2-10x-8}$ for $x \neq -\frac{2}{3}$ or 4

b. $\frac{10x+25}{2x^2-x-15}$ for $x \neq -\frac{5}{2}$ or 3

c. $\frac{(k-4)(2k+1)}{5(2k+1)} \div \frac{(k-3)(k-4)}{10(k-3)}$ for $k \neq 3, 4$ or $-\frac{1}{2}$

3-79. How many solutions does each equation below have? Help (Html5)⇔Help (Java)

a. $4x + 3 = 3x + 3$

b. $3(x - 4) - x = 5 + 2x$

c. $(5x - 2)(x + 4) = 0$

d. $x^2 - 4x + 4 = 0$

3-80. Now David wants to solve the equation $4000x - 8000 = 16{,}000$. Help (Html5)⇔Help (Java)

a. What easier equation could he solve instead that would give him the same solution? (In other words, what equivalent equation has easier numbers to work with?)

b. Justify that your equation in part (a) is equivalent to $4000x - 8000 = 16{,}000$ by showing that they have the same solution.

c. David's last equation to solve is $\frac{x}{100} + \frac{3}{100} = \frac{8}{100}$. Write and solve an equivalent equation with easier numbers that would give him the same answer.

3-81. Solve each of the following inequalities for the given variable. Represent your solutions on a number line. Help (Html5)⇔Help (Java)

a. $5 + 3x < 5$

b. $-3x \geq 8 - x$

3-82. In Lesson 3.2.3 you will focus on multiplying and dividing rational expressions. Recall what you learned about multiplying and dividing fractions in a previous course as you answer the questions below. To help you, the following examples have been provided. Help (Html5)⇔Help (Java)

$$\frac{9}{16} \cdot \frac{4}{6} = \frac{36}{96} = \frac{3}{8}$$

$$\frac{5}{6} \div \frac{20}{12} = \frac{5}{6} \cdot \frac{12}{20} = \frac{60}{120} = \frac{1}{2}$$

a. Without a calculator, multiply $\frac{2}{3} \cdot \frac{9}{14}$ and reduce the result. Then use a calculator to check your answer. Describe your method for multiplying fractions.

b. Without a calculator, divide $\frac{3}{5} \div \frac{12}{25}$ and reduce the result. Then use a calculator to check your answer. Describe your method for dividing fractions.

3-83. Sketch the graph of $y = (x + 2)^3 + 4$. Help (Html5)⇔Help (Java)

a. What is the parent graph of this function? How has the graph of this function been transformed from the parent graph?

b. Rewrite the equation $y = (x + 2)^3 + 4$ without parentheses. Remember the Order of Operations.

c. How would the graph in part (a) differ from the graph of the original equation?

3-84. Sketch the graph of the function $f(x) = 3 \cdot 5^x$. Help (Html5)⇔Help (Java)

a. What is the domain of $f(x)$?

b. Sketch the graph of the geometric sequence $t(n) = 3 \cdot 5n$.

c. What is the difference between $f(x)$ and $t(n)$? Explain completely.