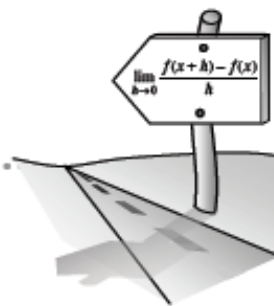# 3.2.2 Which of the tools should I use?

Derivatives Using Multiple Strategies

**3-52.** USING MULTIPLE STRATEGIES TO FIND $f'(x)$

Let $f(x) = x^2 + 4x + 2$. You will find $f'(x)$ using three different methods. Each method should produce the same result.

   a. Use the definition of the derivative.

   b. Use the Power Rule.

   c. Use your graphing calculator: graph the equation $f'(x) = \dfrac{f(x+h) - f(x)}{h}$ for $h = 0.01$. Examine the graph and find the approximate equation of $f'(x)$.

**3-53.** Revisit the Power Rule from problem 3-6. Will the Power Rule work for $f(x) = x^n$ if $n = 0$? When $n$ is negative? What about for non-integer values of $n$? Investigate these conditions with your study team and summarize your results.

**3-54.** Expand the expression $f(x) = (2x - 3)(x^2 + 2)$. Then, use that expression and the Power Rule to find $f'(x)$. Finally, use $f'(x)$ to find the equation of the line tangent to the curve $f(x) = (2x - 3)(x^2 + 2)$ at $x = 3$. Write your equation in point-slope form.

**3-55.** Lazy Lulu wants to find the derivative of $f(x)$ at $x = a$. She used Hana's method to set up the definition of the derivative:

$$\lim_{h \to 0} \frac{((9+h)^2 - 1) - 80}{h}$$

Lulu is lazy and does not want to do algebraic computations. Help Lulu *undo* the definition of the derivative so she can use the Power Rule instead.

   a. What is $f(x)$?

   b. What is $a$?

   c. Avoid the algebra! Use the Power Rule to find $f'(x)$.

   d. What is $f'(a)$?

   e. Find the equation of the tangent *to* $f(x)$ at $x = a$.

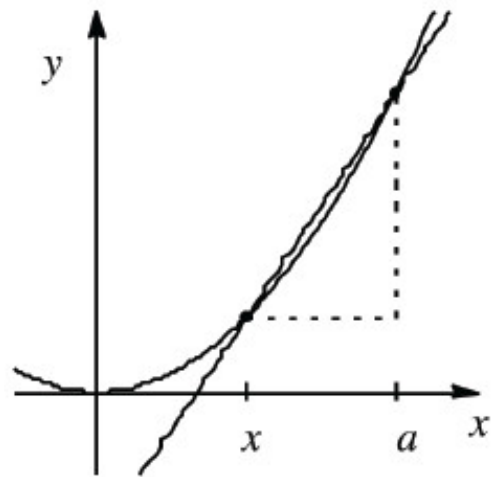**3-56.** ANOTHER DEFINTION OF THE DERIVATIVE: INTRODUCING ANA

Hana, Anah, and Hanah have a stepsister named Ana. Ana also found a method to find the derivative at a point. Her method is a little different from the rest.



Ana's Method:

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

a. Use Ana's method to confirm that the *derivative* of $f(x) = x^2$ at $x = 2$ is 4. Does it work?

b. Use Ana's method to find the *derivative function* of $f(x) = x^2$.

c. What is special about Ana's name?

**3-57.** Graph the function $h(x) = x \sin(2x)$.

a. Can you use the Power Rule? Why or why not?

b. Use the calculator to approximate $h'\left(\frac{\pi}{6}\right)$ using the slope of a small neighboring secant line.

c. Use this approximation to find the equation of the tangent to the curve at $x = \frac{\pi}{6}$.

*Review & Preview*

**3-58.** Irvin measured both his location and velocity while he motorcycled; however, he forgot to label the data and thus mixed up the distance and velocity measurements. Hoping to straighten out the data, he created two graphs. Which graph below represents distance and which graph represents velocity? How do you know?  Help (Html5)⇔Help (Java)

Graph A — Time

Graph B — Time
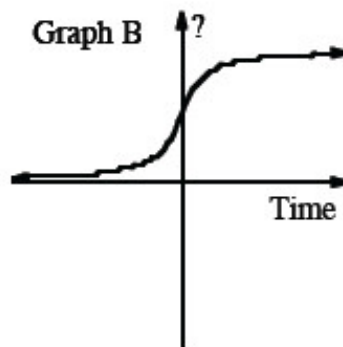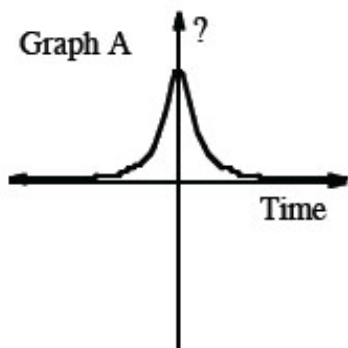
**3-59.** MORE NOTABLE NOTATION FOR THE DERIVATIVE

The use of $\frac{dy}{dx}$ comes from $\frac{\Delta y}{\Delta x}$ which is an expression for slope read as "the change in $y$ over the change in $x$". We use $\Delta$ to represent change. When the change gets smaller and smaller until it is infinitely small (infinitesimal) we use the symbol $d$.

It is useful to think of change when using derivatives. For example $\frac{dh}{dt}$ could represent the change in the height of an object with respect to time. Create expressions using the symbol $d$ that will represent the following instantaneous change statements.   Help (Html5)⇔Help (Java)

   a. The change in the velocity, $v$, with respect to time.

   b. The change in volume, $V$, with respect to the radius, $r$, of a cone.

   c. The change in area, $A$, of a circle with respect to the perimeter, $p$.

**3-60.** Differentiate the following expressions.   Help (Html5)⇔Help (Java)

   a. $f(x) = \frac{9}{x}$

   b. $f(x) = (-3x^7 - 6x)$

   c. $f(x) = (5t^{-4})$

   d. $f(x) = (m)$

**3-61.** Sketch a graph of a function that has the properties listed below. Describe anything special about this function.   Help (Html5)⇔Help (Java)

- $\lim\limits_{x \to -\infty} f(x) = 4$

- $f(1) = -1$

- $f(-x) = -f(x)$

**3-62.** Hanah wrote this derivative function: $f'(x) = \lim\limits_{h \to 0} \dfrac{((x+h)^2 - 3) - ((x-h)^2 - 3)}{2h}$

   a. What is $f(x)$?

   b. What is $f'(x)$? (Note--Avoid the algebra by using the Power Rule.)

   c. Use your slope function to find $f'(0)$ and $f'(1)$.

**3-63.** Answer the following questions using the graph at right, which shows the velocity of a runner over time. The letters $A$ and $B$ represent the areas of the two regions in the diagram.
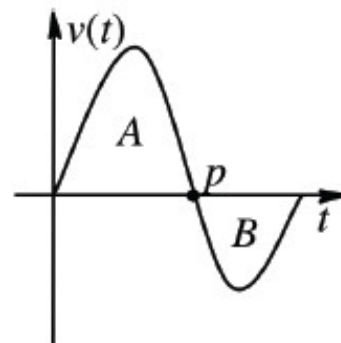


   a. Describe the motion of the runner during the time that is illustrated in the graph.

   b. What is the significance of point $p$

   c. What does the area $A$ represent in this situation? That is, what does it tell you about the runner? What about area $B$?

   d. If $A = 30$ meters and $B = -5$ meters, what does $A + B$ represent in this situation? Why is $B$ negative?

**3-64.** Using the *definition of the derivative* as a limit, show that the derivative of $f(x) = \dfrac{1}{x^2}$ is $f'(x) = -\dfrac{2}{x^3}$.

That is, show algebraically that the following is true:

$$\lim_{h \to 0} \frac{\dfrac{1}{(x+h)^2} - \dfrac{1}{x^2}}{h} = -\frac{2}{x^3}$$

**3-65.** Find the equation of a function that has vertical asymptotes at $x = -3$ and $x = 5$, a hole at $x = 0$ and a horizontal asymptote at $y = 2$. Sketch the function.

**3-66.** Evaluate each limit. If the limit does not exist, say so but also state if $y$ is approaching positive or negative infinity.

   a. $\lim\limits_{x \to 1^+} \dfrac{x^2 + 2x - 3}{x^2 - 2x + 1}$

   b. $\lim\limits_{x \to \infty} \dfrac{x^2 + 6x + 5}{3x^2 + 4}$

   c. $\lim\limits_{x \to 9} \dfrac{\sqrt{x} - 3}{x - 9}$

d.  $\displaystyle\lim_{x \to 2} \frac{x^2+x+1}{\sqrt{x+7}}$

**3-67.** Expand the following trig expressions.   <u>Help (Html5)</u>⇔<u>Help (Java)</u>

a.  $\sin(x + y)$

b.  $\cos(x + y)$

c.  $\sin(x - y)$

d.  $\cos(x - y)$