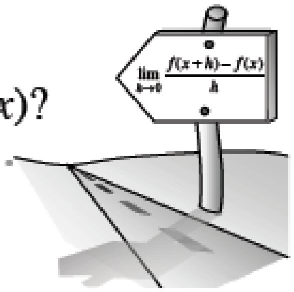
3.2.3 What is the 1000th derivative of sin(x)?



Derivatives of Sine and Cosine

3-68. Summarize all of the tools you have developed so far to find a slope function.

3-69. Use the definition of the derivative to find $f'(x)$ and $g'(x)$ if $f(x) = \sin(x)$ and $g(x) = \cos(x)$. You will need to use the trig identities to simplify your expression.

3-70. DERIVATIVES OF SINE AND COSINE GRAPHICALLY
It appears that the derivatives of sine and cosine are related. What about the second derivatives?

a. Set up five sets of axes, vertically aligned. Scale each x-axis $[-2\pi, 2\pi]$ counting by $\frac{\pi}{6}$ and scale each y-axis $[-2, 2]$ counting by $\frac{1}{2}$. Make sure that the y-axes of each graph are vertically aligned.

b. Sketch as accurately as you can $f(x) = \sin x$ on the top set of axes. Draw bold dots on all maximum and minimum points.

c. On the second set of axes, sketch $f'(x)$ as accurately as you can. Compare the graph of $f(x)$ with $f'(x)$. What does $f(x)$ look like when $f'(x) = 0$?

d. Repeat the process for $f''(x)$, $f'''(x)$, and $f^{iv}(x)$, the second, third and fourth derivatives of $f(x)$. As you work, you might discover shortcuts that will expedite this process. What do you notice about $f^{iv}(x)$?

e. Predict $f^{(20)}(x)$ and $f^{(101)}(x)$.

3-71. Rewrite $y = \frac{1}{x}$ using exponents.

a. Find the slope function y′, or $\frac{dy}{dx}$ algebraically by using the definition of the derivative.

b. Use the Power Rule to confirm your answer to part (a).

3-72. The graph of the equation $y = x^3 - 9x^2 - 16x + 1$ has a slope of 5 at exactly two points.
Find the coordinates of the points. Describe your process.

Review & Preview

3-73. Given $f(x)$ below, find $f'(x)$. Help (Html5)⇔Help (Java)

a. $f(x) = -x^3$

b. $f(x) = \frac{1}{x^2}$

c. $f(x) = \sqrt{2}$

d. $f(x) = 3\sin(x + \pi)$

3-74. Write and then compute a Riemann sum to determine the area $A(f, -4 \le x \le 4)$ where $f(x)$ is the function below. Choose the number of rectangles so that your answer will be a good approximation of the area. What is the name of the shape of which you calculated the area? Confirm the accuracy of the Riemann sum by calculating the area geometrically. Help (Html5)⇔Help (Java)

$$f(x) = \sqrt{16 - x^2}$$

3-75. Differentiate the following expressions. Help (Html5)⇔Help (Java)

a. $\frac{d}{dx}(6^3)$

b. $\frac{d}{dx}\left(\frac{2}{5}x^{15} - \frac{3}{4}x^2\right)$

c. $\frac{d}{dt}(t^{-9})$

d. $\frac{d}{dm}(m^{3/4})$

3-76. Compare three different methods to find a derivative of $f(x) = 2x^3 - x$. Help (Html5)⇔Help (Java)

a. Use the definition of a derivative.

b. Use the Power Rule. Do your answers agree?

c. Use your graphing calculator: graph the equation $f'(x) = \frac{f(x+h) - f(x)}{h}$ for $h = 0.01$. Does the graph match that of $f'(x)$ from part (a)?

3-77. Lazy Lulu is looking at this limit: $\lim_{x\to 3} \frac{x^3+x-30}{x-3}$ and does not want to solve it using algebra. Lulu recognizes this limit as a *definition of the derivative* at a point. She thinks she could use the Power Rule instead. Help (Html5)⇔Help (Java)

a. What variation of the definition of the derivative is this?

b. What is $f(x)$? What is a?

c. Use the Power Rule to find $f'(x)$ and $f'(a)$.

3-78. The position of a ball as a function of time is given by the function below where $s(t)$ is in meters and t is in seconds. Help (Html5)⇔Help (Java)

$$s(t) = \sqrt{t+1}$$

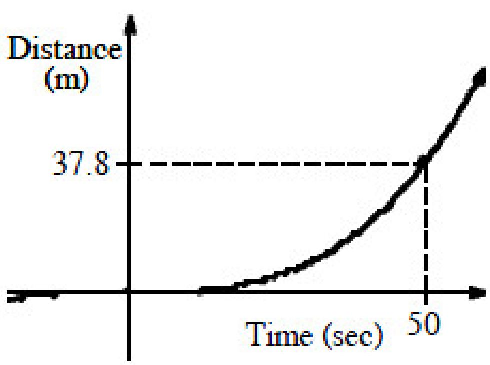
a. Use your calculator to approximate the instantaneous velocity of the ball at 1, 5, 10, and 100 seconds.

b. What do you predict happens to the velocity of the ball after a very long time (i.e. as $t\to\infty$)?

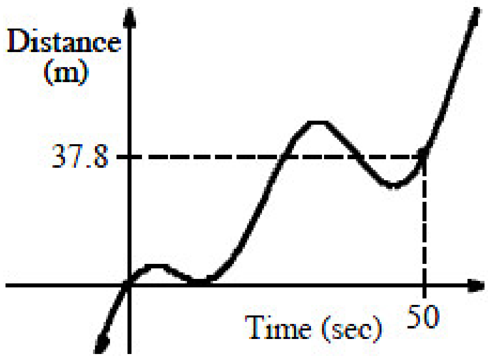
c. What happens to the position of the ball after a very long time, (i.e. what is $s(t)$)? Does this make sense given your answer to part (b)?

3-79. Find the average velocity between 0 and 50 seconds for each of the graphs below. Help (Html5)⇔Help (Java)

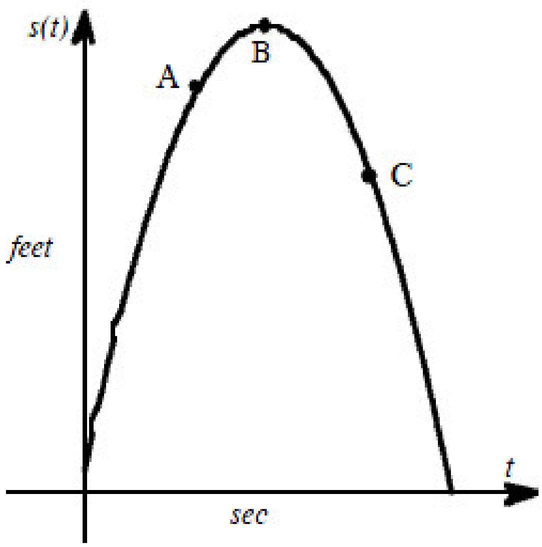
a.



b.



3-80. The graph below shows the height of a soccer ball after being kicked straight up into the air. Help (Html5)⇔Help (Java)



a. Explain why the slope of the tangent at point A will determine the velocity of the ball at that point.

b. At which of the labeled points is the velocity the greatest? How can you tell?

c. At what point does the ball momentarily stop? What is the velocity at this point?

d. The distance from the ground can be described by the function $s(t) = -16t^2 + 76.8t + 5$. Find $v(t) = s'(t)$.

e. Find the instantaneous velocity at $t = 2$, 3, and 4 seconds.

f. What does negative velocity represent in this problem?

3-81. Evaluate each limit. If the limit does not exist, say so but also state if y is approaching positive or negative infinity. Help (Html5)⇔Help (Java)

a. $\lim\limits_{x \to 7^+} \frac{3x-21}{x^2-x-42}$

b. $\lim_{x \to -\infty} \frac{2-3x^2}{2x^2+5x-7}$

c. $\lim_{x \to 0} \frac{\sqrt{x+2}-\sqrt{2}}{x}$

d. $\lim_{x \to \infty} \frac{x^2+3x}{1-x^2-x^3}$