3.2.4 How can I rewrite it?

Adding and Subtracting Rational Expressions

So far in this course you have learned a lot about rational expressions. You have learned how to simplify complex algebraic fractions by factoring the numerators and denominators. You have also learned how to multiply and divide rational expressions. What else is there? Today you will develop a method to add and subtract algebraic fractions.

3-97. With your team, read your directions for Monica's sister from homework problem 3-92. Verify that everyone obtained the same answer and be prepared to share how you added the fractions with the class.

$$\frac{1}{3}+\frac{2}{5}$$

a. Now Monica's sister wants to know *why*? Why does she have to do all of those steps with the common denominator? What is a fraction anyway, and why does adding them have to be so complicated? Draw some pictures or diagrams or make up some situations that will help her to know what fractions like $\frac{1}{3}$ and $\frac{2}{5}$ mean.

b. Now use your ideas from part (a) to show Monica *why* she needs a common denominator to add the two fractions.

3-98. Extend the procedures your class developed for numerical fractions to add these algebraic fractions.

$$\frac{2x}{x-1}+\frac{3}{x+5}$$

3-99. Now add the fractions below. After you have added them, be sure to check to see if the numerator can be factored. You may be able get a simpler answer.

a. $\frac{x}{3x+1}+\frac{2x^2-2}{(x-5)(3x+1)}$

b. $\frac{9-3x}{(x+3)(x-3)}+\frac{2x}{x+3}$

3-100. Examine the expression below.

$$\frac{2x-1}{3x^2+13x+4}+\frac{x+3}{x^2-3x-28}$$

a. With your team, decide how you can alter the expression so that the fractions have a common denominator. Be ready to share your idea with the class.

b. If you have not already do so, add the fractions. Then simplify the result, if possible.

c. Repeat the process to subtract the expressions below. Simplify the result, if possible.

$$\frac{2}{x+4}+\frac{4x-x^2}{x^2-16}$$

3-101. LEARNING LOG

In your Learning Log, explain how to add and subtract rational expressions. Be sure to include an example. Title this entry "Adding and Subtracting Rational Expressions" and include today's date.

METHODS AND MEANINGS

MATH NOTES

Rewriting Rational Expressions

To simplify a rational expression, both the numerator and denominator must be written in factored form. Then look for factors that make a "Giant One"(a form of the number 1) and simplify. Study Examples 1 and 2 below.

Example 1: $\frac{x^2+5x+4}{x^2+x-12}=\frac{(x+4)(x+1)}{(x+4)(x-3)}=1\cdot\frac{x+1}{x-3}=\frac{x+1}{x-3}$ for $x\neq -4$ or 3

Example 2: $\frac{2x-7}{2x^2+3x-35}=\frac{(2x-7)(1)}{(2x-7)(x+5)}=1\cdot\frac{1}{x+5}=\frac{1}{x+5}$ for $x\neq -5$ or $\frac{7}{2}$

Just as you can multiply and divide fractions, you can multiply and divide rational expressions.

Example 3: Multiply $\frac{x^2+6x}{(x+6)^2}\cdot\frac{x^2+7x+6}{x^2-1}$ and symplify for $x\neq -6$ or 1.

After factoring, this expression becomes: $\frac{x(x+6)}{(x+6)(x+6)}\cdot\frac{(x+1)(x+6)}{(x+1)(x-1)}$

After multiplying, reorder the factors: $\frac{(x+6)}{(x+6)}\cdot\frac{(x+6)}{(x+6)}\cdot\frac{x}{(x-1)}\cdot\frac{(x+1)}{(x+1)}$

Since $\frac{(x+6)}{(x+6)}=1$ and $\frac{(x+1)}{(x+1)}=1$, Simplify: $1\cdot 1\frac{x}{(x-1)}\cdot 1\Rightarrow\frac{x}{(x-1)}$

Example 4: Divide $\frac{x^2-4x-5}{x^2-4x+4} \div \frac{x^2-2x-15}{x^2+4x-12}$ and simplify for $x \neq 2, 5, -3$, or -6.

First, change to a multiplication expression: $\frac{x^2-4x-5}{x^2-4x+4} \cdot \frac{x^2+4x-12}{x^2-2x-15}$

Then factor each expression: $\frac{(x-5)(x+1)}{(x-2)(x-2)} \cdot \frac{(x-2)(x+6)}{(x-5)(x+3)}$

After multiplying, reorder the factors: $\frac{(x-5)}{(x-5)} \cdot \frac{(x-2)}{(x-2)} \cdot \frac{(x+1)}{(x-2)} \cdot \frac{(x+6)}{(x+3)}$

Since $\frac{(x-5)}{(x-5)} = 1$ and $\frac{(x-2)}{(x-2)} = 1$, simplify to get: $\frac{(x+1)(x+6)}{(x-2)(x+3)} \Rightarrow \frac{x^2+7x+6}{x^2+x-6}$

Note: From this point forward in the course, you may assume that all values of x that would make a denominator zero are excluded.

Review & Preview

3-102. Estacia wants to learn more about excluded values. Help (Html5)⇔Help (Java)

a. Explain to Estacia why x cannot be 4 in the expression $\frac{x+2}{x-4}$.

b. Find the excluded values of x in each of the expressions of problem 3-99.

c. Create an expression that has the excluded values of $x \neq -6$ and $x \neq \frac{1}{3}$. Be prepared to share your expression to the class.

3-103. Use the methods developed in class to add or subtract the following rational expressions. Be sure to look for factors before trying to determine a common denominator, and simplify your answers, if possible. Help (Html5)⇔Help (Java)

a. $\frac{4x}{x^2-2x-8} + \frac{4}{x-4}$

b. $\frac{16x-12}{4x^2+5x-6} - \frac{3}{x+2}$

3-104. Solve the equations and inequalities below. Check your solutions, if possible. Help (Html5)⇔Help (Java)

a. $|5x+8| \geq -4$

b. $x^2 + x - 20 < 0$

c. $2x^2 - 6x = -5$

d. $\frac{5}{9} - \frac{x}{3} = \frac{4}{9}$

3-105. Simplify the rational expressions below as much as possible. Help (Html5)⇔Help (Java)

a. $\frac{(x-4)^3(2x-1)}{(2x-1)(x-4)^2}$

b. $\frac{7m^2-22m+3}{3m^2-7m-6}$

c. $\frac{(z+2)^9(4z-1)^7}{(z+2)^{10}(4z-1)^5}$

d. $\frac{(x+2)(x^2-6x+9)}{(x-3)(x^2-4)}$

3-106. Lexington High School has an annual growth rate of 4.7%. Three years ago there were 1500 students at the school. Help (Html5)⇔Help (Java)

a. How many students are there now?

b. How many students were there 5 years ago?

c. How many students will there be n years from now?

3-107. Multiply or divide the expressions below. Leave your answers as simplified as possible. Help (Html5)⇔Help (Java)

a. $\frac{(3x-1)(x+7)}{4(2x-5)} \cdot \frac{10(2x-5)}{(4x+1)(x+7)}$

b. $\frac{(m-3)(m+11)}{(2m+5)(m-3)} \div \frac{(4m-3)(m+11)}{(4m-3)(2m+5)}$

c. $\frac{2p^2+5p-12}{2p^2-5p+3} \cdot \frac{p^2+8p-9}{3p^2+10p-8}$

d. $\frac{4x-12}{x^2+3x-10} \div \frac{2x^2-13x+21}{2x^2+3x-35}$

3-108. Graph the function $g(x) = \frac{x+2}{x-1}$ on graph paper and name all x- and y-intercepts. What happens at $x = 1$? Help (Html5)⇔Help (Java)

3-109. If $f(x) = 3x - 9$ and $g(x) = -x^2$, find: Help (Html5)⇔Help (Java)

a. $f(-2)$

b. $g(-2)$

c. x if $f(x) = 0$

d. $g(m)$