3.2.5 Pulling it all together

Creating New Functions

In this lesson you will use all four operations of arithmetic to combine rational expressions. As you work with your team on the problems consider the following questions:

What operation are we using here and what steps will we need to take?

Is it possible to factor the numerators or denominators of the expressions?

How can we use the multiplication property of the "Giant One"?

What values of x must be excluded? How will that affect the graph?

Is our answer a rational expression?

3-110. PULLING IT ALL TOGETHER

You now know how to add, subtract, multiply, and divide rational expressions. Pull this all together by simplifying the following expressions.

a. $\frac{2x^2+x}{(2x+1)^2} - \frac{3}{2x+1}$

b. $\frac{x^2-3x-10}{x^2-4x-5} \div \frac{x^2-7x-18}{2x^2-5x-7}$

c. $\frac{15x-20}{x-5} \cdot \frac{x^2-2x-15}{3x^2+5x-12}$

d. $\frac{4}{2x+3} + \frac{x^2-x-2}{2x^2+5x+3}$

e. $\frac{6x-4}{3x^2-17x+10} - \frac{1}{x^2-2x-15}$

f. $\frac{x^2-x-2}{4x^2-7x-2} \div \frac{x^2-2x-3}{3x^2-8x-3}$

3-111. EXPLORING OPERATIONS WITH RATIONAL FUNCTIONS

What will the graphs of the sum, difference, product or quotient of two rational functions look like? Graphs of rational functions can be very complicated and difficult to interpret using a graphing calculator, so you will work to get a glimpse of some of the simpler outcomes by using two fairly simple rational functions.

$$f(x) = \frac{1}{x-2} \text{ and } g(x) = \frac{1}{x+1}$$

a. Algebraically find $f(x) \cdot g(x)$ and write this as a single function without any parentheses.

b. Graph the simplified function from part (a), and simultaneously graph the function $f(x) \cdot g(x)$. Are the graphs the same? How can you be sure the graphs are the same, and one graph is not just "hiding" out of the window you chose?

c. The graphs should be the same. If they are not, check your algebra (and your input) to see what happened.

d. Algebraically find $f(x)+g(x)$, $f(x)-g(x)$, and $\frac{f(x)}{g(x)}$, and simplify the function.

e. Graph each of your simplified algebraic functions from part (d) simultaneously with the original operation on two functions. Check that the graphs are the same and correct any mistakes if necessary.

f. Along with exploring the shape of the various function operations, you have discovered a way to check you work when simplifying rational expressions. Check you answer to part (a) of problem 3-110.

3-112. Based on your limited experience with rational expressions so far, do you think that the set of all rational expressions is closed for each of the four operations, addition, subtraction, multiplication, and division? If so, what are some reasons you think so, and if not, why not? Discuss this question with your team and be prepared to defend your conjecture.

METHODS AND MEANINGS

MATH NOTES

Adding and Subtracting Rational Expressions

In order to add and subtract fractions, the fractions must have a common denominator. One way to do this is to change each fraction so that the denominator is the **least common multiple** of the denominators. For the example below, the least common multiple of $(x+3)(x+2)$ and $(x+2)$ is $(x+3)(x+2)$.

$$\frac{4}{(x+2)(x+3)}+\frac{2x}{x+2}$$

The denominator of the first fraction already is the least common multiple. To get a common denominator in the second fraction, multiply the fraction by $\frac{(x+3)}{(x+3)}$, a "Giant One" (a form of the number 1).

$$=\frac{4}{(x+2)(x+3)}+\frac{2x}{x+2}\cdot\frac{(x+3)}{(x+3)}$$

Multiply the numerator and denominator of the second term.

$$=\frac{4}{(x+2)(x+3)}+\frac{2x(x+3)}{(x+2)(x+3)}$$

Distribute the numerator, if necessary.

$$=\frac{4}{(x+2)(x+3)}+\frac{2x^2+6x}{(x+2)(x+3)}$$

Add, factor, and simplify the result.

$$= \frac{2x^2+6x+4}{(x+2)(x+3)} = \frac{2(x+1)(x+2)}{(x+2)(x+3)} = \frac{2(x+1)}{(x+3)}$$

Review & Preview

3-113. Add, subtract, multiply, or divide the following rational expressions. Simplify your answers, if possible. Help (Html5)⇔Help (Java)

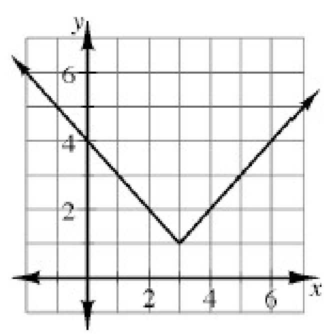
a. $\frac{2x}{3x^2+16x+5} + \frac{10}{3x^2+16x+5}$

b. $\frac{x^2-x-12}{3x^2-11x-4} \cdot \frac{3x^2-20x-7}{x^2-9}$

c. $\frac{2x^2+8x-10}{2x^2+15x+25} \div \frac{4x^2+20x-24}{2x^2+x-10}$

d. $\frac{7}{x+5} - \frac{4-6x}{x^2+10x+25}$

3-114. Examine the graph of $f(x) = |x-3| + 1$ at right. Use the graph to find the values listed below. Help (Html5)⇔Help (Java)



a. $f(3)$

b. $f(0)$

c. $f(4)$

d. $f(-1)$

3-115. Use the graph of $f(x) = |x-3| + 1$ in problem 3-114 to solve the equations and inequalities below. It may be helpful to copy the graph onto graph paper first. Help (Html5)⇔Help (Java)

a. $|x-3| + 1 = 1$

b. $|x-3| + 1 \le 4$

c. $|x-3| + 1 = 3$

d. $|x-3| + 1 > 2$

3-116. This problem is a checkpoint for using function notation and identifying domain and range. It will be referred to as Checkpoint 3B.

Given $g(x) = 2(x+3)^2$, state the domain and range, calculate $g(-5)$ and $g(a+1)$, and then find the value of x when $g(x) = 32$

and when $g(x) = 0$.

Check your answers by referring to the Checkpoint 3B materials.

If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 3B materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do problems like these quickly and easily.

3-117. Solve the quadratic below twice: once by factoring and using the Zero Product Property and once by completing the square. Verify that the solutions match. Help (Html5)⇔Help (Java)

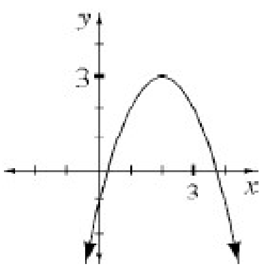
$$x^2 + 14x + 33 = 0$$

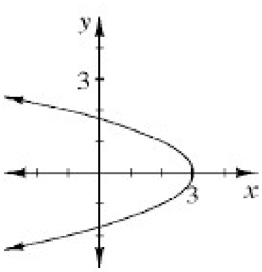
3-118. Match each graph below with its domain. Help (Html5)⇔Help (Java)

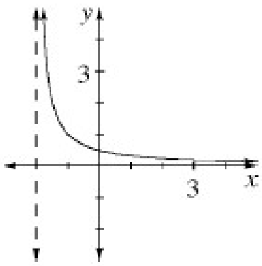
a. D: All values of x

b. D: $x > -2$

c. D: $x \leq 3$

1. 

2. 

3. 

3-119. Graph the two functions below and find all points where they intersect. List all points in the form (x, y). Help (Html5)⇔Help (Java)

$f(x) = x^2 - 3x - 10$

$g(x) = -5x - 7$

3-120. Simplify each expression. Help (Html5)⇔Help (Java)

a. $\frac{1}{x+2} + \frac{3}{x^2-4}$

b. $\frac{3}{2x+4} - \frac{x}{x^2+4x+4}$

c. $\frac{x^2+5x+6}{x^2-9} \cdot \frac{x-3}{x^2+2x}$

d. $\frac{4}{x-2} \div \frac{8}{2-x}$

3-121. Solve $\sqrt{x+2} = 8$ and check your solution. Help (Html5)⇔Help (Java)

3-122. Use each pair of points given below to write a system of equations in $y = mx + b$ form to find the equation of a line that passes through the points. Help (Html5)⇔Help (Java)

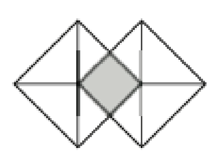
a. (20, 2) and (32, −4)

b. (−3, −17) and (12, −7)

3-123. Phana's garden is 2 meters wide and 5 meters long. She puts a walkway of uniform width around her garden. If the area of the walkway is 30 square meters, what are the outer dimensions of the walkway? Drawing a diagram will help you solve this problem. Help (Html5)⇔Help (Java)

3-124. Leadfoot Lilly was driving 80 miles per hour when she passed a parked highway patrol car. By the time she was half a mile past the spot where the patrol car was parked, the officer was driving after her at 100 miles per hour. If these rates remain constant, how long will it take the officer to catch up to Lilly? Write and solve an equation to represent this situation. Help (Html5)⇔Help (Java)

3-125. Two congruent overlapping squares are shown at right. If a point inside the figure is chosen at random, what is the probability that it will *not* be in the shaded region? Help (Html5)⇔Help (Java)



3-126. Factor each expression completely. Help (Html5)⇔Help (Java)

a. $25x^2 - 1$

b. $5x^3 - 125x$

c. $x^2 + x - 72$

d. $x^3 - 3x^2 - 18x$