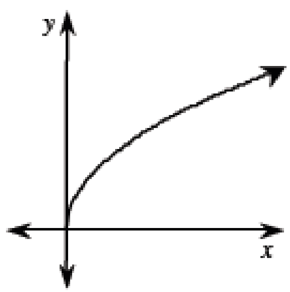
# 3.3.2 Did you notice the curve on that function?

## The Shape of a Curve

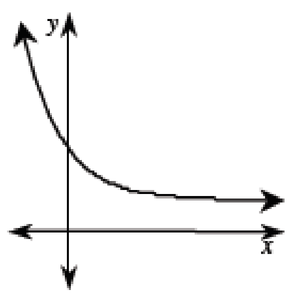
**3-96.** For each graph below, answer the following two questions in complete sentences. Remember that some graphs meet change conditions as $x$ increases.

- As $x$ increases, $f(x)$ increases or decreases ?
- As $x$ increases, the slopes of $f$(x) increase or decrease ?

a.




b.

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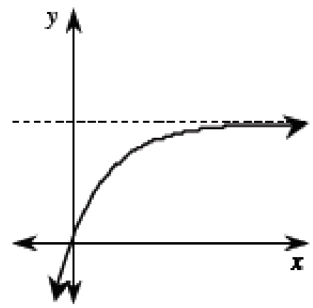


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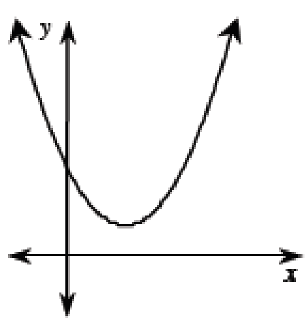
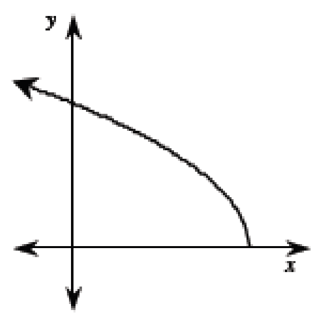
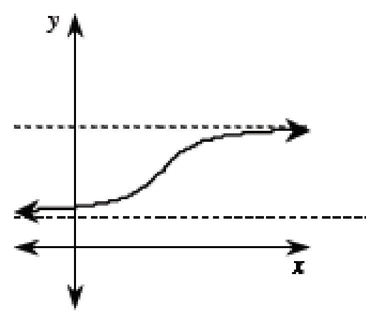



e.




f.




**3-97.** In each part below, sketch a smooth curve in which:

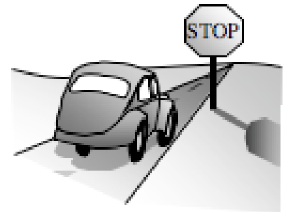
a. The entire curve has a positive, increasing slope.

b. The entire curve has a positive, decreasing slope.

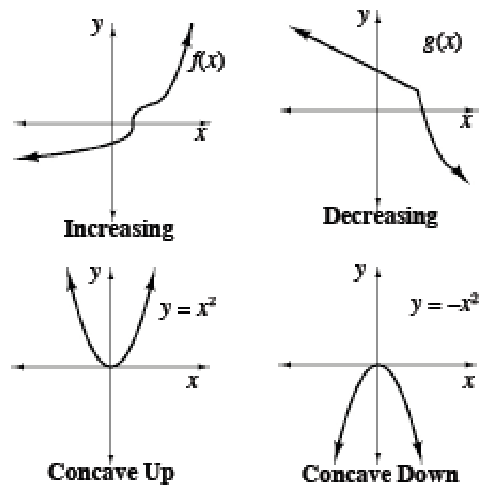
c. The entire curve has a negative, increasing slope.

d. The entire curve has a negative, decreasing slope.

# MATH NOTES



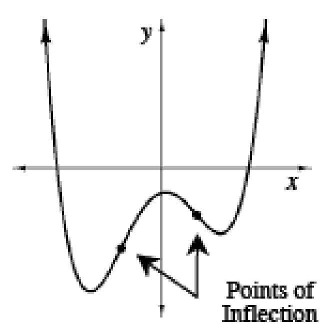

## Curve Analysis




If $y$ increases as $x$ increases, then the function is **increasing.** Likewise, if $y$ is decreasing as $x$ is increasing, the function is **decreasing.** The function below is increasing when $x > 0$ and decreasing when $x < 0$. If a function is increasing or decreasing over its entire domain, the function is **monotonic.**

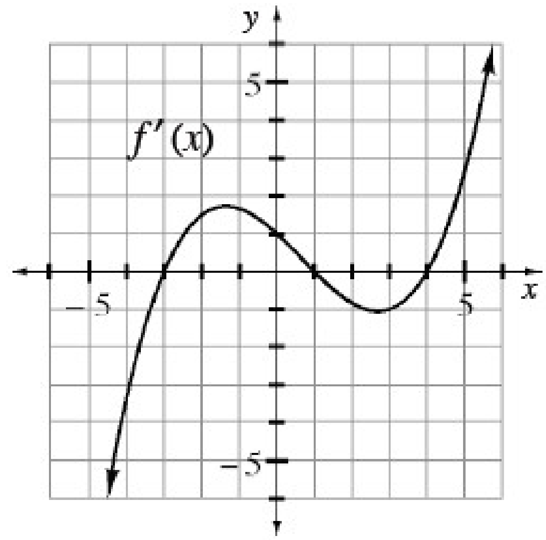
When the slope is increasing over an interval, the function is said to be **concave up** because it is curving upward, like in the curve $y = x^2$ below. However, when the slope is decreasing over an interval, the function is said to be **concave down,** such as with $y = -x^2$.

A point at which concavity changes is called a **point of inflection.** At this point, the curve changes from concave up to concave down or vice versa. A function can also have more than one point of inflection, as shown in the example at right.



**3-98.** Review the graphs from problem 3-96. Which of the curves are concave up, which are concave down, and which are sometimes concave up and sometimes concave down?

**3-99.** The graph at right is the slope function of $f(x)$. Examine the graph carefully as you analyze the following questions.




a. At what $x$-values is $f'(x) = 0$ ? What happens to $f(x)$ at these $x$-values? Thoroughly describe what happens.

b. Use the graph of $f'(x)$ above to identify the parts of the domain on which $f(x)$ is increasing. Explain which graphical clues you used to determine this.

c. At what $x$-value(s) is $f(x)$ at a local minimum (i.e. the lowest point on a local region of a curve)? How can you tell? Explain which graphical clues you used to determine this.

d. Approximate the intervals of $x$ at which $f'(x)$ is increasing. What happens to $f(x)$ at these $x$-values?

**3-100.** Describe the difference between stating, "$f(x)$ is increasing" and "$f'(x)$ is increasing." Which of the two statements indicates that $f'(x)$ is positive

Review & Preview

**3-101.** Draw $f(x)$, given its slope statement.

The slope starts close to zero. When x = −5, the slope increases quickly. Then, at x = 0, the slope begins to decrease quickly until x = 5 when the slope is close to zero again. Help (Html5)⇔Help (Java)

**3-102.** Find $f'(x)$ for each of the following functions. Help (Html5)⇔Help (Java)

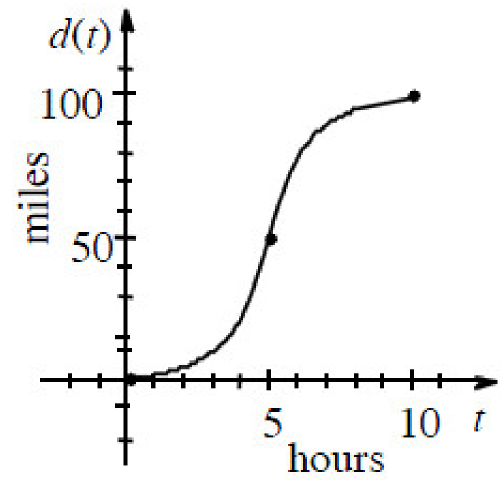
a. $f(x) = \frac{2}{3}(x-5)^3 + x^2$

b. $f(x) = \sqrt[3]{x}$

c. $f(x) = \sin\frac{\pi}{4}$

d. $f(x) = \frac{1}{(x+1)^2}$

**3-103.** The graph at right records the distance a bicyclist travels from Oshkosh to a town 10 miles away. Help (Html5)⇔Help (Java)




a. Describe the velocity of the bicyclist.

b. What was the bike's average velocity

c. Approximate the bike's instantaneous velocity at $t = 5$ hours.

**3-104.** Define $f(x)$ and $g(x)$ so that $h(x) = f(g(x))$ for the following functions given that $f(x) \neq x$ and $g(x) \neq x$. Help (Html5)⇔Help (Java)

a. $h(x) = (2x - 5)^3$

b. $h(x) = \sin(3x - 1)$

c. $h(x) = \sqrt[5]{\tan x}$

**3-105.** If $f'(x) = \cos x$ , find two different possible expressions for $f(x)$. How many solutions are possible? Help (Html5)⇔Help (Java)

**3-106.** Examine the Riemann sum below for the area under $f(x)$. Help (Html5)⇔Help (Java)

$$\sum_{i=0}^{11} \frac{6-3}{12} f\left(3+\frac{6-3}{12} \cdot i\right)$$

a. How many rectangles were used?

b. If the area being approximated is A($f$, $a \leq x \leq b$), what are $a$ and $b$?

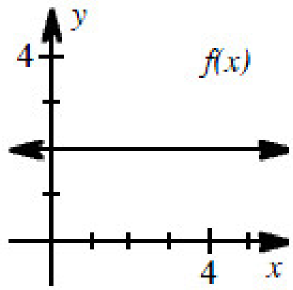
**3-107.** Oliver is trying to find the derivative of $f(x) = -4x^3$ at $x = 2$. He substituted and found that $f(2) = -32$. He then took the derivative and got $f'(2) = 0$. What went wrong? Why didn't this work? Help (Html5)⇔Help (Java)

**3-108.** For each graph below:

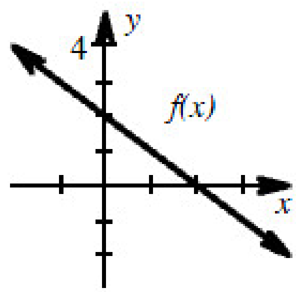
i. Trace $f(x)$ on your paper and write a slope statement for $f(x)$. Help (Html5)⇔Help (Java)

ii. Sketch the graph of $f'(x)$ using a different color.

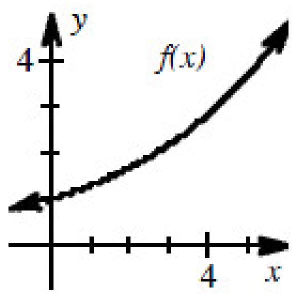
a.




b.




c.




**3-109.** Curves can be labeled with descriptors such as "concave down" and "increasing." On graph paper, graph them and label their respective parts. Use different colors to represent concavity. Help (Html5)⇔Help (Java)

a. $f(x) = 2x^2 + x - 15$

b. $g(x) = x^3 - 12x - 1$

**3-110.** Evaluate each limit. If the limit does not exist, say so but also state if $y$ is approaching positive or negative infinity. Help (Html5)⇔Help (Java)

a. $\lim_{x\to 0} \frac{1-\sqrt{x}}{x-1}$

b. $\lim_{x\to 1} \frac{1-\sqrt{x}}{x-1}$

c. $\lim_{h\to 0} \frac{\sqrt{25+h}-5}{h}$

d. $\lim\limits_{h \to -2} \frac{(h+2)-2}{2}$

e. Two of the problems above can be interpreted as the definition of the derivative at a point. Which are they?