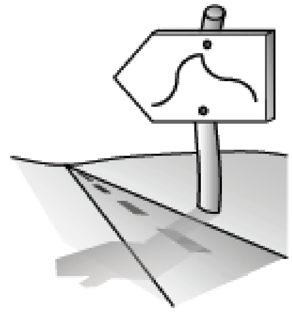
3.3.3 How do I sketch f' and f''?

Curve Sketching: Derivatives



3-111. CURVE ANALYSIS

a. On your Lesson 3.3.3 Resource Page (or a large sheet of graph paper), set up three sets of axes—one above the other so the y-axes are vertically aligned. On each set of axes, let the domain be $-6 \le x \le 6$ and the range be $-5 \le y \le 5$. On the top set of axes, sketch a function $f(x)$ such that:

- $f(x)$ is a continuous, smooth function
- Zeros at $x = -4, -1, 2$ and 5
- Local maximums at $(-3,4)$ and $(2,0)$
- Local minimums at $(0, -2)$ and $(4, -3)$
- Points of inflection at $(-2, 3)$ $(1, -1)$ and $(3, -1)$
- Increasing on $(-\infty, -3) \cup (0, 2) \cup (4, \infty)$
- Decreasing on $(-3, 0) \cup (2, 4)$

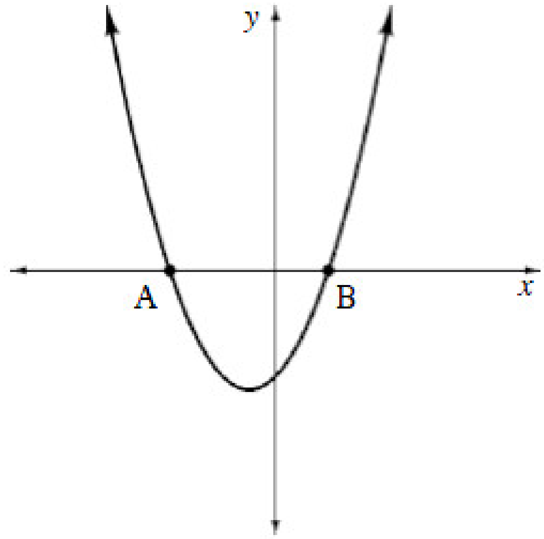
b. Obtain a sheet of stickers or colored markers from your teacher. Choose one color to represent local maximums, another color to represent local minimums, and a third color to represent points of inflection. On the x-axis, use these colors to draw dots at the x-values where the maximum, minimum and point of inflection occur.

c. Using a straightedge, draw long tangent lines to your graph at every integer value from $-6 \le x \le 6$. Sketch slope triangles on each tangent line and use a ruler to approximate Δy and Δx. Record these slopes on a table of data: x-value vs. slope.

d. On the middle set of axes, plot the points from your table of data. Smoothly connect these points to create a continuous graph, be sure to consider what happens as x approaches positive and negative infinity. This graph represents the derivative, f' of your function. Choose a fourth color and label the zeros of $f'(x)$. Compare these points to their corresponding x-values on the graph of $f(x)$. What appears to be significant about these points?

e. Sketch tangent lines on $f'(x)$ and approximate their slopes. Record them on a table of data. Plot that on the third set of axes and connect the points in a smooth and continuous manner, be sure to consider what happens as x approaches positive and negative infinity. This graph is f'', the second derivative of f. Label all zeros. What do you notice?

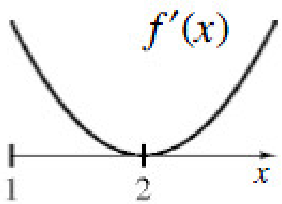
3-112. The graph at right represents $f'(x)$, the first derivative of f.



a. Lyolya looked at this graph and counted two places where f could have a maximum or a minimum value. Where are they? How does she know?

b. Lyolya is unsure how to distinguish between a maximum and a minimum. Help her determine the x-values of the maximum and minimum values of f. Justify your answer.

c. Kat's Controversy: This graph of $f'(x)$, the first derivative of $f(x)$, is confusing to Anabel, Anita and Kat. Anabel thinks there is a local maximum at $x = 2$. Anita thinks there is a local minimum at $x = 2$. And Kat thinks there is a point of inflection at $x = 2$. Who is correct?

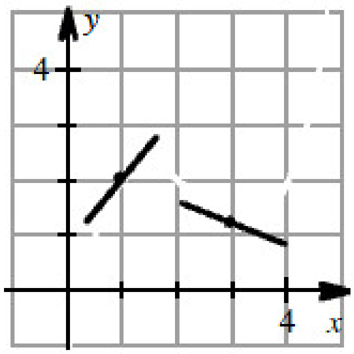


d. If necessary, revise your description of when maximum and minimum values occur in order to address Kat's controversy.

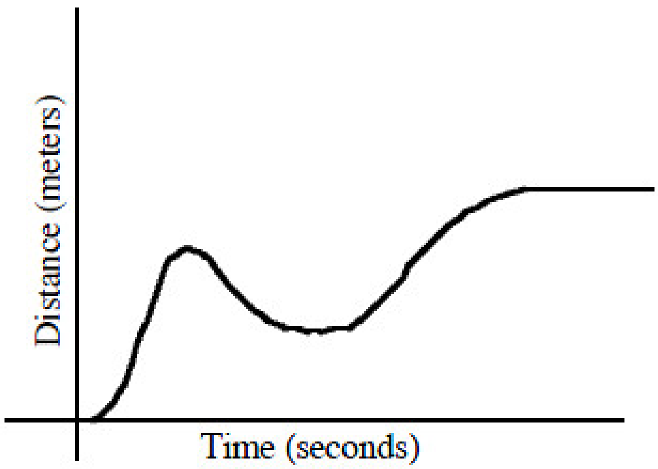
Review & Preview

3-113. Use your observations from problem 3-98 to algebraically verify that $y = x^3 + \frac{3}{2}x^2 - 6x + 2$ is concave up when $x = 0$. Help (Html5)⇔Help (Java)

3-114. Theresa loves tangents! She drew several tangents to a function $g(x)$, and then erased $g(x)$! Trace the graph with the tangents below and draw a possible function $g(x)$. Is there more than one solution? If so, show another using a different color. Help (Html5)⇔Help (Java)



3-115. The graph below shows the distance from a fixed point traveled by a toy car. Use the graph to sketch the velocity of the car. Help (Html5)⇔Help (Java)



3-116. For each function below, write and evaluate a Riemann sum to calculate $A(f, -2 \le x \le 1)$ using 24 left endpoint rectangles. Help (Html5)⇔Help (Java)

a. $f(x)=2^x$

b. $f(x) = \sqrt{x+2}$

3-117. Sketch a function $f(x)$ for which the following is true about its slope function $f'(x)$. Help (Html5)⇔Help (Java)

- For $x < -3$, $f'(x) > 0$ and the slope is increasing.
- For $-3 < x < -1$, $f'(x) > 0$ and the slope is decreasing.
- At $x = -1$, $f'(x) = 0$.
- For $x > -1$, $f'(x) < 0$ and the derivative is decreasing.

3-118. Multiple Choice: If $f'(x) = 6x^2 - 4$, then which of the following could be $f(x)$? Help (Html5)⇔Help

(Java)

a. $f(x) = 2x^3 - 4x + 1$

b. $f(x) = 2x^3 - 4x - 4$

c. $f(x) = 2x^3 - 4x + 4$

d. $f(x) = 2x(x^2 - 2)$

e. All of these

3-119. Compare the limit statements below. What do you notice? Help (Html5)⇔Help (Java)

i. $\lim\limits_{h\to 0} \frac{(x+h+1)(x+h+2)-(x+1)(x+2)}{h}$

ii. $\lim\limits_{h\to 0} \frac{(x+h+1)(x+h+2)-(x-h+1)(x-h+2)}{2h}$

a. What do they have in common? How are they different?

b. Evaluate these limits using any method you choose.

3-120. Compute the limits below. What do you notice? Help (Html5)⇔Help (Java)

i. $\lim\limits_{h\to 0} \frac{(5+h)(6+h)-30}{h}$

ii. $\lim\limits_{x\to 4} \frac{(x+1)(x+2)-30}{x-4}$

3-121. While testing the brakes of a new car, Badru recorded the following speeds traveled (in miles per hour) over time (in seconds). Roughly how far did he travel before stopping? Help (Html5)⇔Help (Java)

t (seconds)	0	1	2	3	4	5	6	7	8	9	10
v(t) (mph)	50	48	46	43	40	37	34	29	23	14	0

3-122. Find all values of x where $f'(x) = 2$ given $f(x) = \frac{1}{3}x^3 - \frac{5}{2}x^2 - 4x + 13$. Help (Html5)⇔Help (Java)

3-123. Evaluate each limit. If the limit does not exist, say so but also state if y is approaching positive or negative infinity. Help (Html5)⇔Help (Java)

a. $\lim_{x \to \infty} \frac{x^3 - x^{-3}}{5x^3 + x^{-3}}$

b. $\lim_{x \to 4} \frac{2 - \sqrt{x}}{x - 4}$

c. $\lim_{x \to 6} \frac{2x^2 - 12x}{x^2 + x - 42}$

d. $\lim_{x \to -\infty} \frac{x^3}{4 + x^2}$