MATH NOTES

Derivative Notations

In the examples you have been working with, for each function $f(x)$ it was possible to find the slope of the graph $y = f(x)$ at any point. These values give us a new function, the **derivative function** (or **slope function).** It has many different notations, all representing the derivative function.

$$f'(x) \text{ read as "} f \text{ prime of } x\text{"}$$

$$\frac{d}{dx} f(x) \text{ read as "the derivative of } f(x) \text{ with respect to } x\text{"}$$

$$\frac{dy}{dx} \text{ read as "} dy, dx \text{" or "} dy \text{ over } dx\text{"}$$

If $y = f(x)$ is the graph of a function, the $f'(x)$ represents the instantaneous rate of change (IROC) $f(x)$ is changing. Note: The derivative at a point on a position graph represents the velocity.

$$f''(x) \text{ read as "} f \text{ double prime of } x\text{"}$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right)$$ read as "the second derivative of f with respect to y"

$$\frac{d^2y}{dx^2}$$ read as "the second derivative of y with respect to x"

3-124. THE SILENT BOARD GAME
Do not talk! Your teacher will draw a large curve on the board. It is your job to label it as completely as possible.
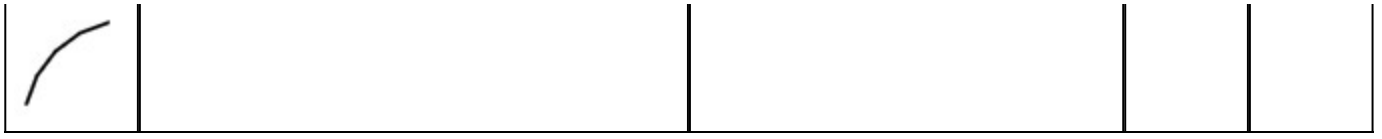
3-125. Walk-a-Wave Challenge
Your teacher will give you a limited amount of time to "walk" one cycle of a perfect sine wave and record it on a motion detector. Use $y = \sin x + 2$. Pay attention to concavity as you walk. When should you move quickly? When should you move slowly? The team with the best graph wins!

3-126. THE SECOND DERIVATIVE

a. If $f'(x)$ represents the rate of change of $f(x)$, then what does $f''(x)$ represent?

b. Since concavity depends on how the slope is *changing*, the concavity must depend on the *slope of the slope*. What does this mean? Explain this in your own words.

c. Examine the curves below and complete the table with the sign of $f(x)$ and $f''(x)$. The first entry has been done for you.

	Increasing or Decreasing? **Walking away from the motion detector?** **or Walking towards the motion detector?**	**Concave Up or Concave Down?** **Getting faster?** **or Getting slower?**	$f'(x)$	$f''(x)$
	Increasing *Away from motion detector*	*Concave Up* *Getting faster*	Positive	Positive

d. How does the increasing or decreasing nature of a curve relate to $f'(x)$? How about $f''(x)$?

e. How does the concavity of a curve relate to $f''(x)$? How about $f'(x)$?

f. Does the direction you walk affect the concavity of the curve? Explain.

3-127. Apply your theories from parts (d) and (e) of problem 3-126 to find where $y = x^3 - 12x$ is increasing and decreasing. Also, find the intervals for which $y = x^3 - 12x$ is concave up and concave down.

3-128. The Math Notes box in Lesson 3.3.2 states that a point of inflection is a point where concavity changes. Since the point of inflection deals with concavity, it must be related to the second derivative.

a. Examine the graph of $y = x^3 - 12x$ and identify where the graph changes concavity. Where is the point of inflection?

b. What is special about y'' at the point of inflection?

3-129. Thoroughly investigate the graph of $f(x) = x^3 - 12x + 4$. Identify all important qualities, such as where the function is increasing, decreasing, concave up and concave down. Also identify point(s) and intercepts and provide graphs of $f'(x)$ and $f''(x)$. Justify all statements <u>graphically</u> and <u>analytically</u>.

—————— Review & Preview ——————

3-130. Summarize your understanding of first and second derivatives. Include information regarding increasing, decreasing, concavity. <u>Help (Html5)</u>⇔<u>Help (Java)</u>

3-131. The graph of $f(x)$ is shown at right. Use the graph to list the following values in ascending (increasing) order: $\frac{f(3)-f(1)}{3-1}$

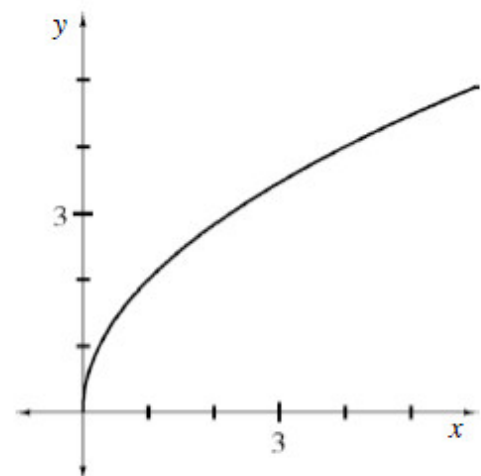0, f'(1), f'(4),
<u>Help (Html5)</u>⇔<u>Help (Java)</u>

3-132. Without your calculator, find the equation of the tangent line to
$$g(z) = \frac{z^7 + 5z^6 - z^3}{z^2} \text{ at } z = 1.$$

<u>Help (Html5)</u>⇔<u>Help (Java)</u>

3-133. If $f'(x) = -6x^{1/2} - \sin x$, find a possible function $f(x)$. Then find another possible function. <u>Help</u>

3-134. For each of the following, find the *second derivative* with respect to x.   Help
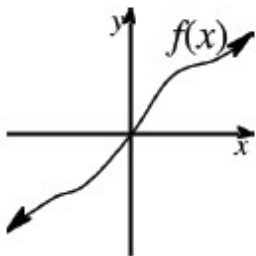
   a. $y = 8x^{99}$

   b. $y = -3\sqrt{x}$

   c. $f(x) = \frac{2}{3}x - 6x^{-2}$

   d. $f(x) = 7 - 2\cos x$

3-135. If a function $f(x)$ is increasing, such as the one below, what must be true about $f'(x)$? Use this idea to determine algebraically where $f(x) = 3x^2 - 3x + 1$ is increasing.


3-136. Write a Riemann sum to estimate $A(f, 2 \le x \le 3)$ for $f(x) = 9x - 2$ with 20 left endpoint rectangles. Then compute the actual area geometrically and calculate the percent error.

3-137. For the absolute value function shown below, something interesting happens to the tangent of the function at $x = 2$. Can you draw more than one tangent at $x = 2$? Do you think this function has any valid tangents at $x = 2$?
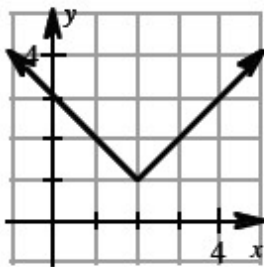
3-138. For each graph below:

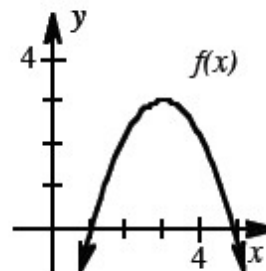   i. Trace $f(x)$ on your paper and write a slope statement for $f(x)$.

   ii. Sketch the graph of $f'(x)$ using a different color.
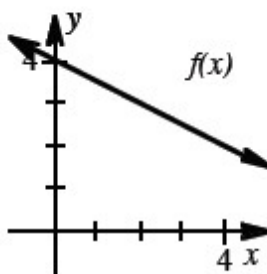
a.
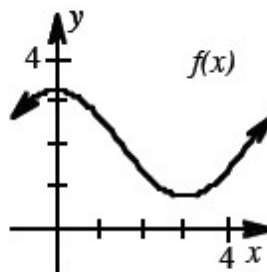
b.

c.

3-139. While riding a roller coaster, the velocity in meters per second on a certain segment of track is represented by the function $v(t) = -8t$.   Help (Html5)⇔Help (Java)

a. Sketch a graph of $v(t)$ for $0 \le t \le 4$. What is the speed of the car at $t = 4$?

b. Assuming that the car is 100 meters above ground at $t = 0$, where is the car at $t = 4$? How far is this drop?

3-140. Evaluate each limit. If the limit does not exist, say so but also state if y is approaching positive or negative infinity.   Help (Html5)⇔Help (Java)

a. $\lim\limits_{x \to -2} (2x^2 - 6x + 5)^2$

b. $\lim\limits_{x \to \infty} \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$

c. $\lim\limits_{x \to 2^-} \dfrac{(x-1)(x-2)}{x+1}$

d. $\lim\limits_{x \to 2^-} \dfrac{x+2}{(x-1)(x-2)}$