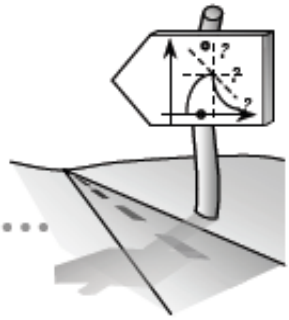
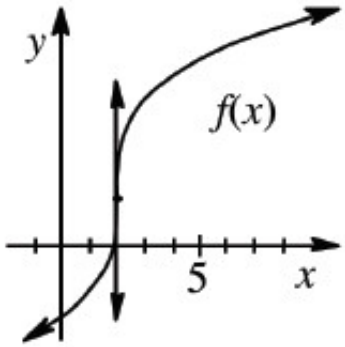
3.4.1 Do your points match? Do your slopes match?



Conditions for Differentiability

3-141. Review the definition of a derivative. Explain why we need to use a limit to find the slope of a tangent. Be clear in your explanation and use a diagram to help demonstrate the limit.

3-142. A line tangent to $f(x)$ at $x = 2$ is shown below. Does the slope of the tangent exist? Does $f'(2)$ exist? Why or why not?

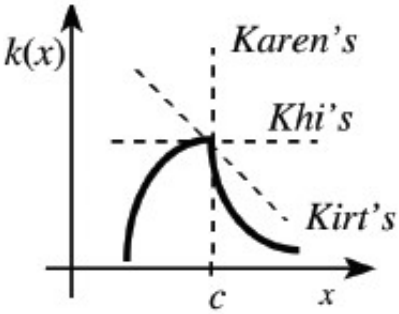


3-143. On your graphing calculator, graph $f(x) = \sqrt[3]{x}$.

a. Describe what happens to the slope of $f(x)$ at $x = 0$.

b. Use the Power Rule to find an equation for $f'(x)$ and use it to find $f'(0)$. Explain what happened. Does this confirm what you see in the graph?

3-144. HELP!
Koy's team needs your help to settle a dispute. For the function $k(x)$ shown in bold below, each of her team members drew a different tangent at $x = c$.

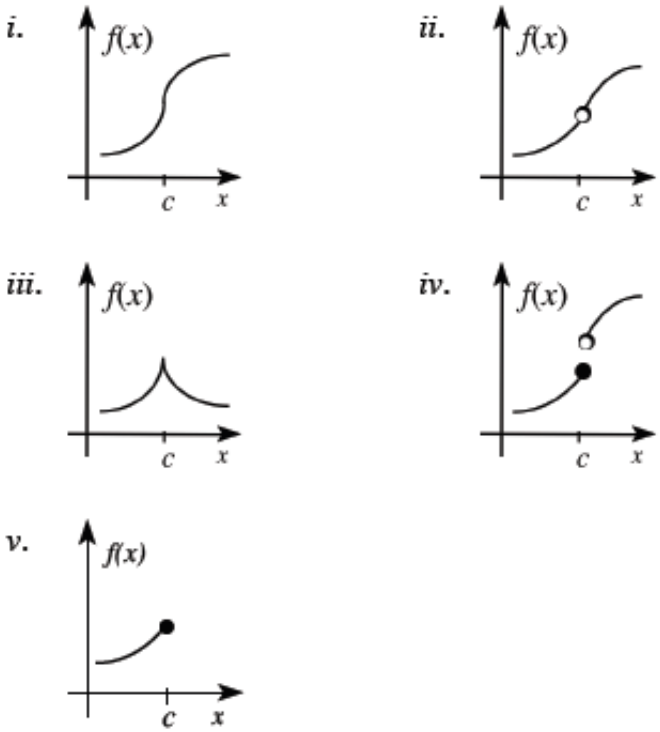


Khi reasoned that if you follow x as it approaches c from the left side, then his tangent makes the most sense. Karen says hers makes sense because hers is tangent to the right side of the curve at $x = c$. Kirt argues that his is the best solution for both sides.

Koy does not think any of them are correct because the slope of a tangent is a limit, and a limit cannot exist if both sides do not agree.

Which team member is correct and why?

3-145. If exactly one tangent can be drawn to a function $f(x)$ at $x = c$ and if the slope of that tangent exists then the function is **differentiable** at that point $f(c)$. The graphs below show examples of functions that are not differentiable at $f(c)$, each for a different reason.



a. With your team, discuss why the slope of a tangent does not exist at f(c) in each case.

b. Sketch $f'(x)$ for each. Pay close attention for what happens at $x = c$.

c. In your own words, describe when a function is **differentiable** and when it is **non-differentiable.**

3-146. ANTIDERIVATIVES

a. If $f'(x) = 8x^3 - 10x - 5$, find a possible function *for* $f(x)$. This is called an **antiderivative** of $f'(x)$ since its derivative is the starting function.

b. Explain why there is always more than one antiderivative.

MATH NOTES

Antiderivatives

An **antiderivative** of a function $f(x)$ is a new function $F(x)$ whose derivative is $f(x)$.

That is, $\frac{d}{dx}F(x) = f(x)$.

The antiderivative of $f'(x)$ is $f(x)$. However, for the antiderivative of a function $f(x)$, we write a capital letter $F(x)$. For example, we write the antiderivative of $g(x)$ as $G(x)$.

Since there are an infinite number of antiderivatives that are different only in the constant term, called a "family," we add a constant "C" to represent all of them. This is known as the **general antiderivative** (or simply the antiderivative). For example:

If $F(x) = 5x^3 + 3x^2 + C$, then $F(x) = 5x^3 + 3x^2 + 5$ and $F(x) = 5x^3 + 3x^2 - 9$ are both antiderivatives of $f(x)$. This is why we write $F(x) = 5x^3 + 3x^2 + C$.

3-147. Find the general antiderivative $F(x)$ for each function below. Test your solution by verifying that $F'(x) = f(x)$. Help (Html5) ⇔ Help (Java)

a. $f(x) = -6x^5 + 12x^2$

b. $f(x) = 3 \cos x$

3-148. Sketch the graph of a function continuous over all real numbers that satisfies *both* of the following properties. Help (Html5)⇔Help (Java)

- $\lim_{x \to 2^-} f'(x) = 0$
- $\lim_{x \to 2^+} f'(x) = \infty$

3-149. Review problem 3-109. Then on graph paper, graph the functions below and label their parts with increasing, decreasing, concave up, concave down, and point of inflection. Then use the first and second derivatives to find the intervals of increasing, decreasing and concavity. Use different colors to represent concavity. Help (Html5)⇔Help (Java)

a. $f(x) = x^2 + 5x - 6$

b. $g(x) = \frac{1}{3}x^3 + 3x^2 - x + 5$

3-150. Use the definition of a derivative to find y' if $y = \frac{3}{4}x^2 - 11x + 34$. Confirm your solution with the Power Rule. Help (Html5)⇔Help (Java)

3-151. State the domain and the range of the functions below. Help (Html5)⇔Help (Java)

a. $y = \frac{x^2 - x - 6}{x + 1}$

b. $y = \frac{x^2 - x - 6}{x + 2}$

3-152. Find $z'(x)$ if $z(x) = 3x^2 + 5x + 1$. Then, find the equation of the tangent in point slope form at $x = -2$. Help (Html5)⇔Help (Java)

3-153. Sketch a continuous curve which meets all the criteria: Help (Html5)⇔Help (Java)

- $f'(x) > 0$ for all x
- $f(x)$ is concave down.
- $f(2) = 1$
- $f'(2) = \frac{1}{2}$

a. How many roots do $f(x)$ have?

b. What can you say about the location of the root(s)?

c. Find $\lim_{x \to -\infty} f(x)$.

d. Is it possible that $f'(1) = 1$? $f'(1) = \frac{1}{4}$? For each case, explain why or why not.

3-154. Sketch a graph and find the equation of the line tangent to the curve $y = 3\sqrt[3]{x-2}$ at $x = 3$. Then, find another point on the curve with the same slope as the tangent. Help (Html5)⇔Help (Java)

3-155. Re-examine your graph of $y = 3\sqrt[3]{x-2}$ from problem 3-154. Help (Html5)⇔Help (Java)

a. As accurately as possible, draw a tangent to y at $x = 2$. Estimate its slope.

b. According to your derivative, y', what is the slope of the tangent at $x = 2$?