Differentiability of Specific Functions
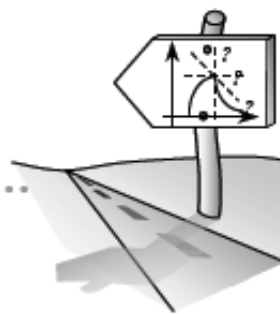
3-170. FUNKY FUNCTIONS, Part Two

One of the reasons we need to analyze functions analytically is that the graphs can at sometimes be misleading. When viewed with a standard window, the graph of $f(x) = 2 + (0.1 - |x|)^2$ can look differentiable at $x = 0$ when it is not! If the curve is continuous and has a local linearization at a point $(c, f(c))$ then the function is differentiable at $(c, f(c))$. Examine the graphs of the following "funky functions" and their graphs to determine if they are differentiable at $x = c$.

a. $f(x) = \frac{\sin x}{x}$, $c = 0$

b. $f(x) = \begin{cases} |x|^x & \text{for } x \neq 0 \\ 1 & \text{for } x = 0 \end{cases}$

c. $f(x) = |x^3 + 0.125|$, $c = -0.5$

3-171. Graph the function defined by $g(r) = \begin{cases} 0 & \text{for } -2 \leq r \leq 2 \\ r^3 + 2r^2 - 4r - 8 & \text{otherwise} \end{cases}$.

a. Is g continuous at $r = 2$? Explain your answer.

b. Do you think g is differentiable at $r = 2$? Explain your answer.

3-172. Sketch the graph of a function defined for all real numbers that satisfies all of the following properties. (There are lots of possible answers.)

- $f(0) = -1$

- $f(x)$ is not differentiable at $x = 2$.

- $f(x)$ is decreasing for all $x \neq 2$.

3-173. Compare how distance and velocity are related with these two scenarios:

a. A ball is thrown down a ramp so that the distance it has traveled in feet at time t is $d(t) = 6t^2 + 2t$. Without

your calculator, find the velocity, $d\,'(t)$, at time $t = 1$, 3, and 10 seconds. Explain what concepts of calculus you applied in order to solve this problem.

b. When a football is kicked from the ground straight up into the air its velocity, measured in feet per second, is $v(t) = -32t + 80$. Sketch a graph of distance and velocity and find the maximum height obtained the ball. Explain what calculus concepts you applied to solve this problem.

c. Both (a) and (b) involve distance and velocity. However, each required a different method or approach. Describe the relationship between distance and velocity, as well as the derivative and area under a curve.

―――――――― Review & Preview ――――――――
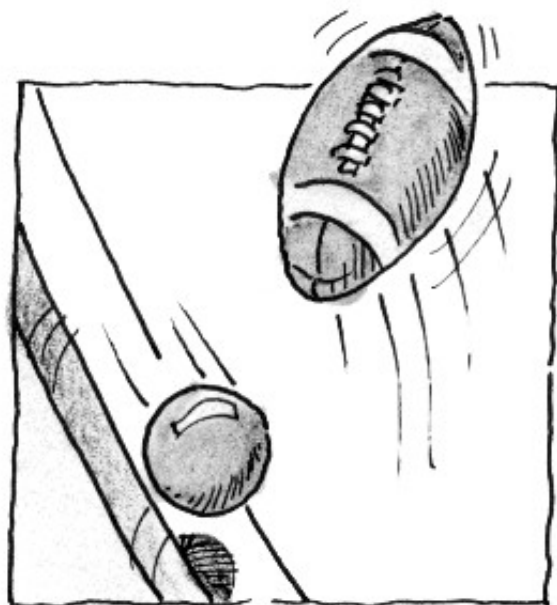
3-174. Sketch the graph of $f(x) = x(x - 1)$, and use this graph to sketch the graph of the slope function, $f\,'(x)$. Help (Html5)⟺Help (Java)

3-175. Use the *definition of a derivative* as a limit to find the slope function, $f\,'(x)$, of $f(x) = 2x^2 - 3x + 4$. Confirm your slope function with the Power Rule. Then use your slope function to find $f\,'(3)$ and $f\,'(-2)$.   Help (Html5)⟺Help (Java)

3-176. Find the following limits quickly. (Hint: Review your solution for problem 3-92 first!)   Help (Html5)⟺Help (Java)

a. $\lim\limits_{h \to 0} \dfrac{(x+h)^5 - x^5}{h}$

b. $\lim\limits_{h \to 0} \dfrac{2\sqrt{x+h} - 2\sqrt{x}}{h}$

3-177. Write a Riemann sum to estimate $A(g,\ 0 \le x \le 2)$ for $g(x) = -x^2 + 4$ using n left endpoint rectangles. Help (Html5)⟺Help (Java)

a. Then calculate for $n = 20$.

b. How can you use your result to estimate $A(g, -2 \le x \le 0)$? $A(g, -2 \le x \le 2)$?

3-178. Find the end behavior function for the following functions: <u>Help (Html5)</u>⇔<u>Help (Java)</u>

a. $f(x) = \frac{2x^2 - 3x + 1}{x + 2}$

b. $g(x) = \frac{1}{x} + \sin x$

c. $h(x) = \frac{\sin x}{x}$

3-179. Find the inverse of each of the following functions. Assuming that no domains are restricted, which of the following has an *inverse function*? How do you know?   <u>Help (Html5)</u>⇔<u>Help (Java)</u>

a. $f(x) = -10x + 8$

b. $g(x) = (x + 4)^2$

c. $f(x) = x^3 + 2$

d. $h(x) = 3 \sin x$

3-180. Let f be the function given by $f(x) = x^3 - 3x^2 - 24x + k$ where k is a constant. On what intervals is f increasing?   <u>Help (Html5)</u>⇔<u>Help (Java)</u>

3-181.
Find the volume of the flag below rotated about the pole. In a complete sentence, describe the rotated shape.
   <u>Help (Html5)</u>⇔<u>Help (Java)</u>