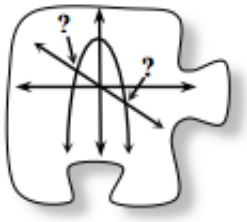
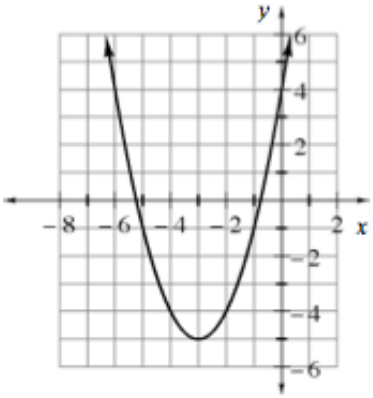
4.1.1 How can I solve?



Strategies for Solving Equations

Today you will have the opportunity to solve challenging equations. As you work with your team, the goal of this section is for you to apply your strategies for solving equations to other types of equations. You will be challenged to use multiple approaches and to write clear explanations to show your understanding.

4-1. SOLVING GRAPHICALLY



One of the big questions of Chapter 2 was how to the find special points of a function. For example, you now have the skills to look at an equation of a parabola written in graphing form and name its vertex quickly. But what about the locations of other points on the parabola? Consider the graph of $y = (x + 3)^2 - 5$ at right.

a. How many solutions does the equation $y = (x + 3)^2 - 5$ have? How is this shown on the graph?

b. Use the graph to solve the equation $(x + 3)^2 - 5 = 4$. How did the graph help you solve the equation?

4-2. ALGEBRAIC STRATEGIES

The graph in problem 4-1 was useful to solve an equation like $(x + 3)^2 - 5 = 4$. But what if you do not have an accurate graph? And what can you do when the solution is not on a grid point or is off your graph?

Your Task: Solve the equation below algebraically (that is, using the equation only and without a graph) in at least three different ways. The "Discussion Points" below are provided to help you get started. Be ready to share your strategies with the class.

$$(x + 3)^2 - 5 = 4$$

Discussion Points

What algebraic strategies might be useful?

What makes this equation look challenging? How can we make the equation simpler?

How can we be sure that our strategy helps us find *all* possible solutions?

4-3. Three strategies your class or team may have used in problem 4-2 are **Rewriting** (using algebra to write a new equivalent equation that is easier to solve), **Looking Inside** (reasoning about the value of the expression inside the function or parentheses), and **Undoing**(reversing or doing the opposite of an operation; for example,

taking the square root to eliminate squaring). These strategies and others will be useful throughout the rest of this course. Examine how each of these strategies can be used to solve the equation below by completing parts (a) through (f).

$$\frac{x-5}{4}+\frac{2}{5}=\frac{9}{10}$$

a. Ernie decided to multiply both sides of the equation by 20 so that his equation becomes $5(x - 5) + 8 = 18$. Which strategy did Ernie use? How can you tell?

b. Elle took Ernie's equation and decided to subtract 8 from both sides to get $5(x - 5) = 10$. Which strategy did Elle use?

c. Eric looked at Elle's equation and said, "*I can tell that $(x - 5)$ must equal 2 because $5 \cdot 2 = 10$. Therefore, if $x - 5 = 2$, then x must be 7.*" What strategy did Eric use?

d. How many solutions does the function $y=\frac{x-5}{4}+\frac{2}{5}$ have? How can you use the graph of $y=\frac{x-5}{4}+\frac{2}{5}$ on your graphing calculator to check your solution to $\frac{x-5}{4}+\frac{2}{5}=\frac{9}{10}$? Where did you look on the graph?

e. How can you use the table for $y=\frac{x-5}{4}+\frac{2}{5}$ on your graphing calculator to check your answer? Where did you look on the table?

f. Use the strategies from parts (a) through (c) in a different way to solve $\frac{x-5}{4}+\frac{2}{5}=\frac{9}{10}$. Did you get the same result?

4-4. Solve each equation below, if possible, using any strategy. Check with your teammates to see what strategies they chose. Be sure to check your solutions.

a. $4|8x-2|=8$

b. $3\sqrt{4x-8}+9=15$

c. $(x - 3)^2 - 2 = -5$

d. $(2y - 3)(y - 2) = -12y + 18$

e. $\frac{5}{x}+\frac{1}{3x}=\frac{4x}{3}$

f. $|3-7x|=-6$

g. $\frac{6w-1}{5}-3w=\frac{12w-16}{15}$

h. $(x + 2)^2 + 4(x + 2) - 5 = 0$

4-5. Some of the solutions from the previous problem can quickly be checked with a graph or table on the graphing calculator. Check the answers for those problems with your graphing calculator.

4-6. LEARNING LOG

Create a Learning Log entry about all of the solving strategies you saw today. For each strategy, show an example and explain which types of equations work best with that strategy. Title this entry "Strategies for Solving Equations" and label it with today's date.

Review & Preview

4-7. Solve $(x - 2)^2 - 3 = 1$ graphically. That is, graph $y = (x - 2)^2 - 3$ and $y = 1$ on the same set of axes and find the x-value(s) of any points of intersection. Then use algebraic strategies to solve the equation and verify that your graphical solutions are correct. Help (Html5)⇔Help (Java)

4-8. Solve each equation below. Think about Rewriting, Looking Inside, or Undoing to simplify the process. Help (Html5)⇔Help (Java)

a. $2(x-1)^2 + 7 = 39$

b. $7(\sqrt{m+1} - 3) = 21$

c. $\frac{x}{2} + \frac{x}{3} = \frac{5x+2}{6}$

d. $-7 + (\frac{4x+2}{2}) = 8$

4-9. Find the equation of the line that passes through (0, 2) and (5, 2). Then complete parts (a) and (b) below. Help (Html5)⇔Help (Java)

a. What is the equation of the x-axis?

b. What is the equation of the y-axis?

4-10. Solve the system of equations shown below. Help (Html5)⇔Help (Java)

$$2x + 6y = 10$$
$$x = 8 - 3y$$

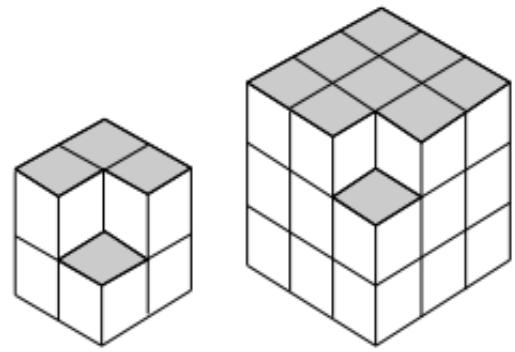
a. Describe what happened when you tried to solve the system.

b. Draw the graph of the system.

c. How does the graph of the system explain what happened with the equations? Make your answer as clear and thorough as possible.

4-11. Classify the triangle with vertices $A(3, 2)$, $B(-2, 0)$, and $C(-1, 4)$ by finding the length of each side. Be sure to consider all possible triangle types. Include sufficient evidence to support your conclusion. Help (Html5)⇔Help (Java)

4-12. Examine the figures at right, and then visualize the figure for $n = 4$. Help (Html5)⇔Help (Java)



a. How many cubes are in the figure for $n = 4$?

b. How many cubes are in the figure for $n = 1$?

c. Find the general equation for the number of cubes for any n. Verify your formula with the cases of $n = 1$ and $n = 5$.

d. Is the sequence arithmetic, geometric, or neither? Explain your reasoning.

4-13. Simplify each of the expressions below. Express your answers as simply as possible. Help (Html5)⇔Help (Java)

a. $\frac{5x^2-11x+2}{x^2+8x+16} \cdot \frac{x^2+10x+24}{10x^2+13x-3}$

b. $\frac{6x+3}{2x-3} \div \frac{3x^2-12x-15}{2x^2-x-3}$

c. $\frac{5m+18}{m+3} + \frac{4m+9}{m+3}$

d. $\frac{3a^2+a-1}{a^2-2a+1} - \frac{2a^2-a+2}{a^2-2a+1}$

4-14. The graph of a line and an exponential can intersect twice, once, or not at all. Describe the possible number of intersections for each of the following pairs of graphs. Your solution to each part should include all of the possibilities and a quickly sketched example of each one. Help (Html5)⇔Help (Java)

a. A line and a parabola

b. Two different parabolas

c. A parabola and a circle

d. A parabola and the hyperbola $y = \frac{1}{x}$