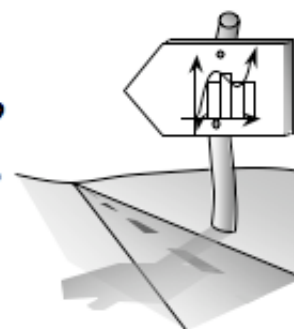


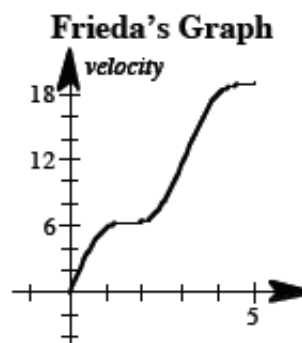
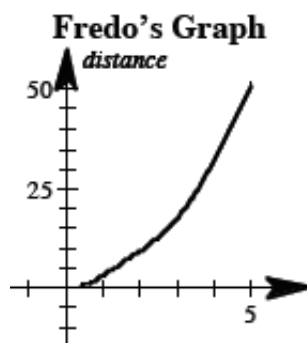
4.1.1 How do we calculate the exact area?

Definite Integrals



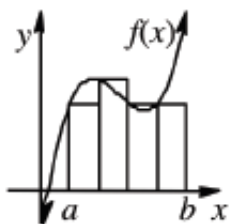
4-1. THE RETURN OF FREDO AND FRIEDA

Examine these new velocity and distance graphs from Fredo and Frieda. Summarize how Fredo's data is reflected in Frieda's graph and how Frieda's data is reflected in Fredo's graph. You may want to review your results from problems Lesson 1.5.1.

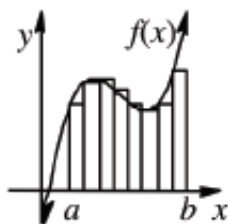


- Since each student's data is confirmed by the others, using a derivative (to find the slope of Fredo's graph) must be linked to using an integral (to find the area under Frieda's graph). Explain the forward and backward nature of the link between slope and area.
- Recall that the slope of a secant line can be used to find the average velocity (approximate slope) at any point on Fredo's curve. How can the exact slope of a curve at a point be found?
- Rectangles can be used to approximate the area under Frieda's curve in order to find the distance. Theorize on how the exact area under a curve can be determined.

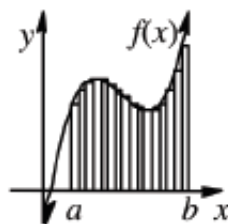
4-2. Regardless of what value of n is chosen, a Riemann sum can only *approximate* the area under $f(x)$ on $[a, b]$ because the rectangles either add extra area or miss some area. Some values of n give a better approximation than others. Examine the graphs below and write down your observations.



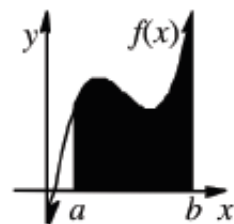
4 rectangles



8 rectangles



16 rectangles



200 rectangles

- How could we get an exact area? Using complete sentences, describe your ideas thoroughly.
- The width of each rectangle is found by $\frac{b-a}{n}$ and is represented by Δx . What happens to Δx as we use more rectangles?

4-3. CALCULATING EXACT AREA

- Using a Riemann sum, write an expression to represent the exact area under $f(x)$ on $[a, b]$.
- Will Δx ever equal 0? Why or why not?
- What happens to the area of each individual rectangle as $n \rightarrow \infty$?
- If the area is composed of rectangles with areas that are approaching zero, why doesn't the overall area approach zero?

MATH NOTES



Definite Integrals

A Riemann sum is a convenient way to approximate the area under a curve using rectangles. However, in order to get an exact area, we would need an infinite number of rectangles! Since we cannot substitute infinity into our Riemann sum, we take a limit as *the number of rectangles approaches infinity*. This "limit of a Riemann sum," shown below, has another name: a **definite integral**. Its symbol represents the "S" of "summe," the German word for sum.

$$A(f, a \leq x \leq b) = \lim_{n \rightarrow \infty} \sum_{i=0}^{n-1} \Delta x \cdot f(a + \Delta x \cdot i) = \int_a^b f(x) dx$$

Therefore, the exact area for $A(x^2 + 1, -2 \leq x \leq 3)$ is usually represented by

$$\int_{-2}^3 (x^2 + 1) dx$$

In the integral above, -2 is the **lower bound**, 3 is the **upper bound**, and the expression $x^2 + 1$ is called the **integrand**. Note that all of our rectangles in this definition are of equal width. It is possible to define the definite integral when the widths of the intervals are different so long as all of the widths approach zero.

4-4. Examine the general form of a definite integral, $\int_a^b f(x) dx$, as shown in the Math Note above.

- What do the upper " b ," and lower " a ," bounds of the integral represent?
- What happened to Δx ?
- Explain why it is important to remember that we are multiplying $(x^2 + 1)$ by dx .

4-5. For the following integrals, draw a sketch of the function and shade the appropriate region. Describe the region, and then find the area without your calculator.

a. $\int_0^{2\pi} \sin(2t) dt$

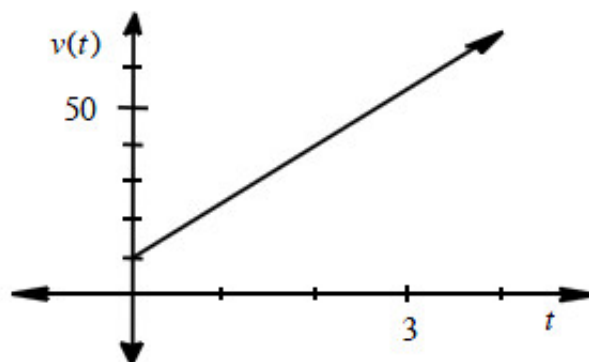
b. $\int_{-2}^3 \left(\frac{1}{2}x + 3\right) dx$

c. $\int_0^6 4 dx$

d. $\int_4^4 (3k^2) dk$

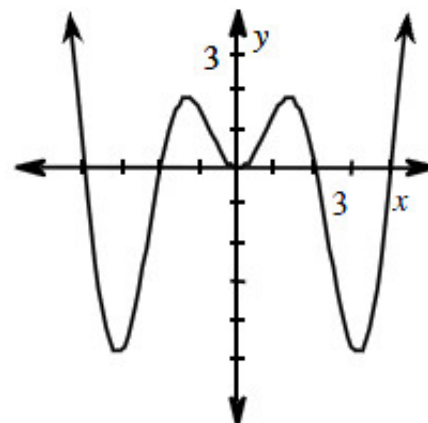


4-6. While driving to work, Mr. Matlack's velocity was $v(t) = 15t + 10$, where t is hours and $v(t)$ is miles per hour. Determine how far Mr. Matlack lives from school if it takes him: [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)



- 1 hour to get to work.
- 4 hours to get to work.
- $\frac{1}{2}$ hour to get to work.
- t hours to get to work.

4-7. Examine $f(x)$ graphed at right. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)



- Is $f(x)$ even, odd, or neither?
- If $\int_0^2 f(x) dx = 10$, what is $\int_{-2}^2 f(x) dx$? Explain.
- If $\int_0^3 f(x) dx = -2$ and $\int_0^2 f(x) dx = 10$ find $\int_2^3 f(x) dx$. Explain.
- If you know that $\int_0^3 f(x) dx$ and $\int_{-2}^2 f(x) dx$, how can you find $\int_2^3 f(x) dx$? Justify your process with a diagram, if necessary.

4-8. For each function, find its slope function, $f'(x)$. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

- $f(x) = \frac{2}{5}x^{-2} - 4x$
- $f(x) = -2\sqrt{x}$
- $f(x) = 6 \cos x$
- $f(x) = \frac{4x^2+4}{x^2+1}$

4-9. Write a Riemann sum that finds $A(f, -8 \leq x \leq 8)$ for $f(x) = 2x^{1/3} + 1$, using the number of rectangles given below. Then evaluate with the summation feature of your calculator. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

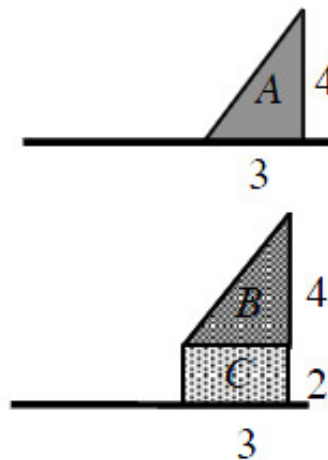
a. 16 rectangles

b. 64 rectangles

4-10. Use the Power Rule to find $f'(x)$ for $f(x) = (x + 4)(x - 3)$ by first expanding $f(x)$. Then, find the equation of the line tangent to $f(x)$ at $x = 3$. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

4-11. Write a Riemann sum that approximates $A(f, 2 \leq x \leq 5)$ with n rectangles for $f(x) = 3x + 5$. Then find the approximate area using at least four different values of n . For what value of n is your approximation most accurate and why? [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

4-12. Khi thinks that when the flags below are rotated, the (volume of A) + (volume of C) = (volume of $(B + C)$).



Decide if he is correct. If he is incorrect, explain the error in his logic.