Lesson 4.1.1

4-1. See below:

b. Take a limit of the slope of the secants as the two point converge. Emphasize that without the limit, we could not find a slope at a single point.

c. Again, take the limit of the sum of the areas of the rectangles as the width of each rectangle goes to zero.

4-2. See below:

a. Take the limit of the sum of areas as the number of rectangles $\rightarrow \infty$.

b. Gets smaller, approaching 0**.**

4-3. See below:

a. Students should write an expression of the form: $\lim\limits_{n \to \infty} \sum\limits_{i=0}^{n-1} \left(\frac{b-a}{n}\right) \cdot f(a + \frac{b-a}{n} \cdot i)$

b. No; The limit predicts a sum as n approaches infinity but n cannot equal infinity.

c. Each gets smaller, approaching 0.

d. Answers vary.

4-4. See below:

a. The starting and ending points for the area region.

b. It became dx, which can be thought of as an infinitely thin width.

c. $f(x)$ represents the heights of all rectangles and dx represents the bases.

4-5. See below:

a. 0; The regions above and below the x-axis are equal.

b. 16.25; The region is a trapezoid.

c. 24; The region is a rectangle.

d. 0, The region has a width of 0.

Review & Preview

4-6. See below:

a. 17.5

b. 160

c. 6.875

d. $7.5t^2 + 10t$

4-7. See below:

a. even

b. $2 \cdot 10 = 20$

c. $-2 - 10 = -12$

d. $\int_0^3 f(x)\,dx - \frac{1}{2}\int_{-2}^{2} f(x)\,dx = \int_2^3 f(x)\,dx$

4-8. See below:

a. $\frac{-4}{5}x^{-3} - 4$

b. $-x^{-1/2}$

c. $-6 \sin x$

d. 0

4-9. See below:

a. $\sum_{i=0}^{15}\left[f(-8+i)^{1/3}+1\right] = 12$

b. $\sum_{i=0}^{63}\left[\frac{1}{4}\cdot\left(f(-8+\frac{1}{4}\cdot i)^{1/3}+1\right)\right] = 15.5$

4-10. $f'(x) = 2x + 1$, $y = 7(x - 3)$

4-11. Answers should lie near 46.5; using more rectangles is more accurate. $\sum_{i=0}^{n-1} \frac{3}{n}\left(3\left(\frac{3i}{n}+2\right)+5\right)$

4-12. No, the further the region is from the "pole," the more volume it sweeps; Vol $A = 16\pi$ un^3, Vol $C = 12\pi$ un^3, Vol $(B + C) = 52\pi$ un^3