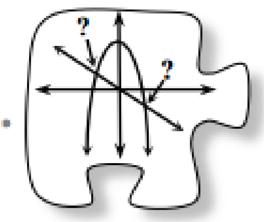
4.1.2 How can I use a graph to solve?

Solving Equations and Systems Graphically



In the previous lesson, you used and named three algebraic methods to solve different kinds of equations. In today's lesson, you will again solve equations, but this time you will use your understanding of graphs, as well as your algebra skills, to solve the equations and to verify your results.

4-15. In problem 4-1, you used a graph to solve an equation. In what other ways can a graph be a useful solution tool? Consider this question as you solve the equation $\sqrt{2x+3}=x$ by completing parts (a) through (d) below.

a. Use algebraic strategies to solve $\sqrt{2x+3}=x$. How many solutions did you find? Which strategies did you use?

b. In thinking about $\sqrt{2x+3}=x$, Miranda wrote down $y=\sqrt{2x+3}$ and $y=x$. How many solutions does $y=\sqrt{2x+3}$ have? How many solutions does $y=x$ have?

c. Miranda said, "*I'll graph both the functions* $y=\sqrt{2x+3}$ *and* $y=x$ *to check the solutions from part (a).*" How will graphing help her find the solution?

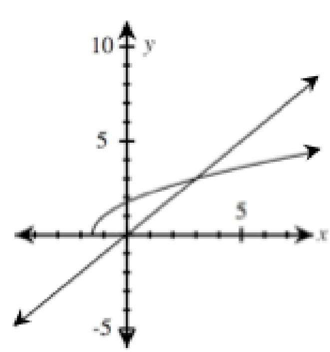
d. Miranda looked at the graph on her graphing calculator and said "*I think something is wrong.*" What happened? Graph the system on your graphing calculator and find the intersection(s) of the functions. How many solutions does this equation have?

4-16. When a result from an equation-solving process does not make the original equation true, it is called an **extraneous solution**. It is not a solution of the equation, even though it is a result when solving algebraically.

Check your two solutions from part (a) of problem 4-15 algebraically.

4-17. The fact that extraneous solutions can arise after following straightforward solving techniques makes it especially important to check your solutions!

But why did the extraneous solution appear in this problem? Examine the graph of the system of equations $y=\sqrt{2x+3}$ and $y=x$, shown at right. Where would an extraneous solution $x=-1$ appear on the graph? Why do the graphs not intersect at that point? Explain.



4-18. After solving the equation $2x^2 + 5x - 3 = x^2 + 4x + 3$, Gustav got called to the office and left his team. When his teammates examined his graphing calculator to try to find out how he found his solution, they only saw the graph of $y = x^2 + x - 6$. Consider this situation as you answer the questions below.

a. How many solutions do you predict $2x^2 + 5x - 3 = x^2 + 4x + 3$ will have?

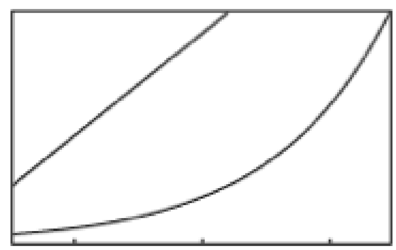
b. Solve $2x^2 + 5x - 3 = x^2 + 4x + 3$ algebraically.

c. Where did Gustav get the equation $y = x^2 + x - 6$? How many solutions will $y = x^2 + x - 6$ have?

d. How can you see the solutions to $2x^2 + 5x - 3 = x^2 + 4x + 3$ in the graph of $y = x^2 + x - 6$? Explain why this makes sense.

e. Maiya solved $2x^2 + 5x - 3 = x^2 + 4x + 3$ by graphing a system of equations and looking for the points of intersection. What equations do you think she used? Graph these equations on your graphing calculator and explain where the solutions to the equation exist on the graph.

4-19. Karen could not figure out how to solve $20x + 1 = 3^x$ algebraically, so she decided to use her graphing calculator. However, after she finished entering the equations $y = 20x + 1$ and $y = 3^x$, she got the graph shown right. After studying the graph, Karen suspects there are no solutions to $20x + 1 = 3^x$.



a. What do you think? If there are solutions, find them and prove that they are solutions. If there are no solutions, demonstrate that there cannot be a solution.

b. What should solutions to the equation, $20x + 1 = 3^x$, look like? In other words, will solutions be a single number, or should they be the coordinates of a point? Explain.

c. Elana started to solve first by subtracting 1 from both sides of her equation. So when she graphed her system later, she used the equations $y = 20x$ and $y = 3^x - 1$. Should she get the same solutions? Test your conclusion with your graphing calculator.

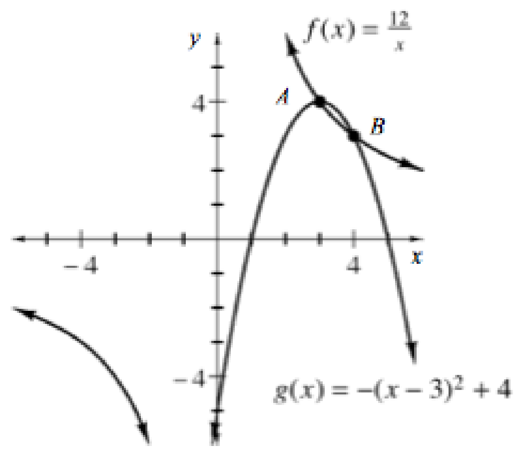
d. Discuss with your team why Karen could not solve the system algebraically. What do you think?

4-20. Jack was working on solving an equation and he graphed the functions $f(x) = \frac{12}{x}$ and $g(x) = -(x - 3)^2 + 4$, as shown below.

a. What equation was Jack solving?

b. Use points A and B to solve the equation you wrote in part (a).

c. Are there any other solutions to this same equation that are represented by neither point A nor point B? If so, show that these other solutions make your equation true.



4-21. LEARNING LOG

What does the solution to an equation mean? Do you have any new ideas about solutions that you did not have before? Create a Learning Log entry that explains the meaning of a solution in as many ways as possible. Title this entry "The Meaning of Solution, Part 1" (Parts 2 and 3 will be coming later) and label it with today's date.

4-22. Solve $(x - 3)^2 - 2 = x + 1$ graphically. Is there more than one way to do this? Explain. Help (Html5)⇔Help (Java)

4-23. Graph a system of equations to solve $2|x-4|-3=\frac{2}{3}x-3$. Show your solutions clearly on your graph. Help (Html5)⇔Help (Java)

4-24. Solve each of the following equations using any method. Be sure to check your solutions. Help (Html5)⇔Help (Java)

a. $-3\sqrt{2x-5}+7=-8$

b. $2|3x+4|-10=12$

4-25. Ted needs to find the point of intersection for the lines $y = 18x - 30$ and $y = -22x + 50$. He takes out a piece of graph paper and then realizes that he can solve this problem without graphing. Explain how Ted is going to accomplish this, and then find the point of intersection. Help (Html5)⇔Help (Java)

4-26. Consider the arithmetic sequence 2, $a - b$, $a + b$, 35 , ... Find a and b. Help (Html5)⇔Help (Java)

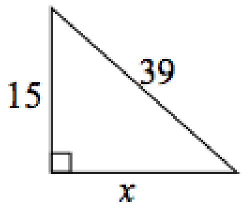
4-27. Solve the following equations. Be sure to check your answers for any extraneous solutions. Help (Html5)⇔Help (Java)

a. $\sqrt{2x-1}-x=-8$

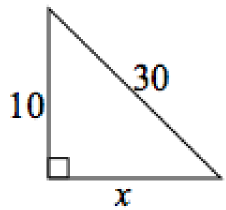
b. $\sqrt{2x-1}-x=0$

4-28. Find the value of x. Help (Html5)⇔Help (Java)

a.

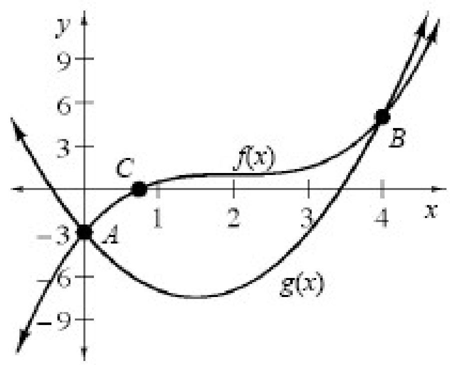


b.



4-29. Solve $3x-1=2^x$ graphically. Could you solve this equation algebraically? Explain. Help (Html5)⇔Help (Java)

4-30. Consider the graphs of $f(x)=\frac{1}{2}(x-2)^3+1$ and $g(x)=2x^2-6x-3$ at right. Help (Html5)⇔Help (Java)



a. Write an equation that you could solve using points A and B. What are the solutions to your equation? Substitute them into your equation to show that they work.

b. Are there any solutions to the equation in part (a) that do not appear on the graph? Explain.

c. Write an equation that you could solve using point C. What does the solution to your equation appear to be? Again, substitute your solution into the equation. How close was your estimate?

d. What are the domains and ranges of $f(x)$ and $g(x)$?

4-31. Solve each of the following equations using any method. Help (Html5)⇔Help (Java)

a. $2(x+3)^2-5=-5$

b. $3(x-2)^2+6=9$

c. $|2x-5|-6=15$

d. $3\sqrt{5x-2}+1=7$

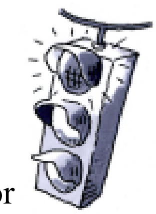
4-32. Solve each of the following equations for the indicated variable. Help (Html5)⇔Help (Java)

a. $5x - 3y = 12$ for y

b. $F = \frac{Gm_1m_2}{r^2}$ for m_2

c. $E = \frac{1}{2}mv^2$ for m

d. $(x - 4)^2 + (y - 1)^2 = 10$ for y



4-33. Paul states that $(a + b)^2$ is equivalent to $a^2 + b^2$. Joyce thinks that something is missing. Help Joyce show Paul that the two expressions are not equivalent. Explain using at least two different approaches: diagrams, algebra, numbers, or words. Help (Html5)⇔Help (Java)

4-34. Graph each of the following equations. (Keep the graphs handy, because you will need them for your homework for Lesson 4.1.3.) Help (Html5)⇔Help (Java)

a. $y = |x|$

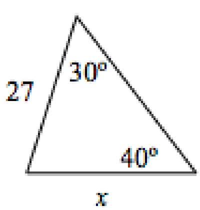
b. $|y| = x$

c. How are the two graphs similar? How are they different?

d. What are the domain and range of each relation?

4-35. Find the value of x. Help (Html5)⇔Help (Java)

a.



b.