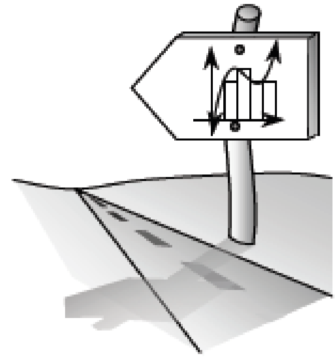
4.1.2 How accurate is a Riemann sum?



Numerical Cases of Definite Integrals

4-13. Use the numerical integration feature of your graphing calculator to calculate the integrals below. Then, for each region, evaluate a Riemann sum to estimate the area with 20 rectangles. Finally, compare these results.

a. $\int_{-\pi}^{\pi/2} 3\sin(2x)\,dx$

b. $\int_{-5}^{5} (x^2 - 3)\,dx$

4-14. *Without a calculator*, graph and shade the region represented by the integrals below. Then, rewrite this expression using only oneintegral. Verify with your calculator that the sum of the three integrals is equal to the single integral.

$$\int_{1}^{3} (9x-2)\,dx + \int_{3}^{8} (9x-2)\,dx + \int_{-2}^{1} (9x-2)\,dx$$

4-15. Draw and shade the region representing $\int_{5}^{0} 4x\,dx$.

a. Evaluate the integral geometrically and then verify your result using the numerical integration feature of your graphing calculator. What happened?

b. Why is this different than $\int_{0}^{5} 4x\,dx$

4-16. Does $\int_{a}^{b} f(x)\,dx = \int_{b}^{a} f(x)\,dx$? Why or why not?

Examine the limit of a Riemann sum, $\lim\limits_{n\to\infty} \sum\limits_{i=0}^{n-1} \frac{b-a}{n} \cdot f(a + \frac{b-a}{n} \cdot i)$ as you answer this question.

4-17. COMBINING REGIONS

Rewrite the following expressions as a single integral.

a. $\int_1^6 f(x)\,dx - \int_3^6 f(x)\,dx$

b. $\int_3^{10} f(x)\,dx + \int_9^3 f(x)\,dx$

c. $\int_c^d f(x)\,dx + \int_e^c f(x)\,dx$

d. $\int_a^{x+h} f(t)\,dt - \int_a^x f(t)\,dt$

e. When can we combine multiple regions? When can we rewrite them? Use the above examples to justify your answer.

Review & Preview

4-18. Evaluate the following definite integrals. Help (Html5)⇔Help (Java)

a. $-\int_0^3 x\,dx$

b. $\int_0^3 (-x)\,dx$

c. $\int_3^0 x\,dx$

d. $-\int_3^0 (-x)\,dx$

4-19. For each $f(x)$, find its general antiderivative, $F(x)$. Help (Html5)⇔Help (Java)

a. $f(x) = -2$

b. $f(x) = \frac{3}{2}x^{-1/2}$

c. $f(x) = -3x^2 + 6x$

d. $f(x) = 2(x + 3)$

4-20. Differentiate each function below. That is, find its slope function, $f'(x)$. Help (Html5)⇔Help (Java)

a. $f(x) = 6(x - 2)^3$

b. $f(x) = 2 \sin x$

c. $f(x) = (x + 5)(2x - 1)$

d. $f(x) = \frac{x^3 - 6x^2 + 2x}{x}$

4-21. Describe a "slope function" in complete sentences. What is its purpose? Give some examples of functions and their slope functions. Help (Html5)⇔Help (Java)

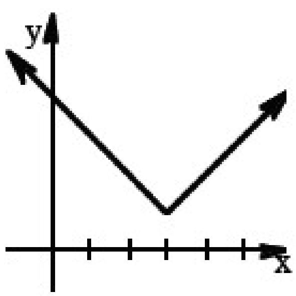
4-22. The value of $\int_0^{\pi} x \cdot \sin x \, dx$ is π. Help (Html5)⇔Help (Java)

a. Without a calculator, find: $2\int_0^{\pi} x \cdot \sin x \, dx$ and $\int_{-\pi}^{\pi} x \cdot \sin x \, dx$.

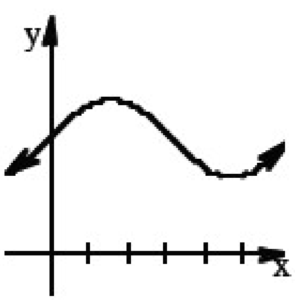
b. Use a graph to justify your conclusion.

4-23. For parts (a) and (b) below, trace $f(x)$ on your paper. Then, using a different color pencil or highlighter, sketch the graph of $y = f'(x)$ for the function given. Help (Html5)⇔Help (Java)

a.



b.

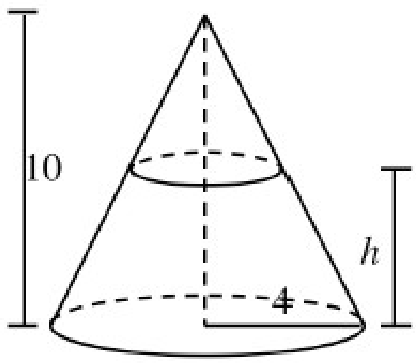


4-24. Determine the following limits quickly. Help (Html5)⇔Help (Java)

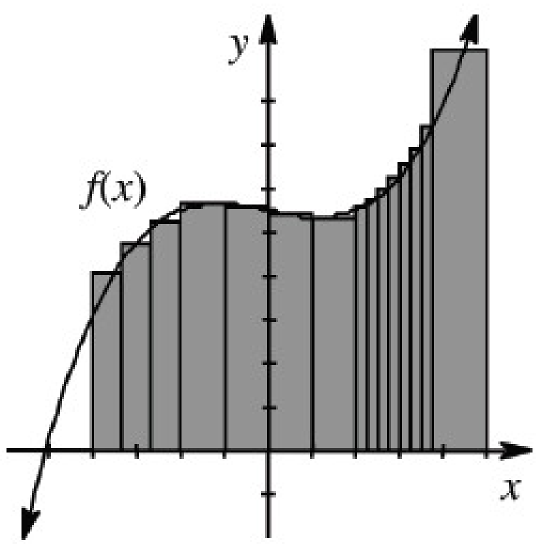
a. $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$

b. $\lim_{h \to 0} \frac{(1+h)^2 - 1}{h}$

4-25. A cone has a height of 10 inches and a radius of 4 inches. If a plane cuts the cone h inches above the base of the cone, write an expression for the area of the circular cross-section. Help (Html5)⇔Help (Java)



4-26. Georg Friedrich Bernhard Riemann (1826-1866) is the person who formulated the modern definition of an integral. He decided that it was not absolutely necessary that each rectangle have the same width. They do not even need to be the same type (i.e. they all do not need to be endpoint or midpoint rectangles).



For example, examine the rectangles used above to estimate the area under $f(x)$. Will they still give a good estimate of area even though the rectangles do not have the same width? Help (Html5)⇔Help (Java)