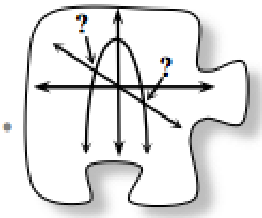
4.1.3 How many solutions are there?



Finding Multiple Solutions to Systems of Equations

You have used many different solving strategies to find solutions of equations with one variable both algebraically and graphically. You have also worked with systems of two equations with two variables. In this lesson, you will use your algebraic and graphing tools to determine the number of solutions that various systems have and to determine the meaning of those solutions.

4-36. Solve each system of equations below without graphing. For each one, explain what the solution (or lack thereof) tells you about the graph of the system.

a. $y = -3x + 5$
   $y = -3x - 1$

b. $y = \frac{1}{2}x^2 + 1$
   $y = 2x - 1$

c. $y^2 = x$
   $y = x - 2$

d. $4x - 2y = 10$
   $y = 2x - 5$

4-37. Now consider the system shown below.

$$x^2 + y^2 = 25$$
$$y = x^2 - 13$$

a. How many solutions do you expect this system to have? Explain how you made your prediction.

b. Solve this system by graphing. How many solutions did you find? Was your prediction in part (a) correct?

c. Find a way to combine these equations to create a new equation so that the only variable is x. Then find another way to combine $x^2 + y^2 = 25$ and $y = x^2 - 13$ to form a different equation that contains only the variable y. Which of these equations would be easier to solve? Why?

d. If you have not already done so, solve one of the combined equations from part (c). If solving becomes too difficult, you may want to switch to the other combined equation.

4-38. In problem 4-37, you analyzed the system shown below.

$$x^2 + y^2 = 25$$
$$y = x^2 - 13$$

a. What minor adjustments can you make to an equation (or both equations) in this system so that the new system has no solutions? Have each member of your team find a different way to alter the system. **Justify** that your system has no solution algebraically. Also, be ready to share your strategies for changing the system along with your justification with the class.

b. Work with your team to alter the system three more times so that the new systems have 3, 2, or 1 solution. For each new system that your team creates, solve the system algebraically to study how the algebraic solution helps indicate how many solutions will be possible. Be prepared to explain what different situations occur during solving that result in a different number of solutions.

4-39. LEARNING LOG

Look over your work from today. Name all of the strategies you used to solve systems of equations. Which strategies were most useful for solving linear systems? What about non-linear systems? Write a Learning Log entry describing your ideas you about solving systems. Title this entry “Finding Solutions to Systems” and label it with today's date.

Review & Preview

4-40. Solve each of the following systems algebraically. What do the solutions tell you about each system? Visualizing the graphs may help with your description. Help (Html5)⇔Help (Java)

a. $y = 3x - 5$
$y = -2x - 15$

b. $y - 7 = -2x$
$4x + 2y = 14$

c. $y = 2(x + 3)^2 - 5$
$y = 14x + 17$

d. $y = 3(x - 2)^2 + 3$
$y = 6x - 12$

4-41. Solve each equation below. Think about rewriting, looking inside, or undoing to simplify the process. Help (Html5)⇔Help (Java)

a. $3(y + 1)^2 - 5 = 43$

b. $\sqrt{1-4x} = 10$

c. $\frac{6y-1}{y} - 3 = 2$

d. $\sqrt[3]{1-2x}=3$

4-42. This problem is a checkpoint for writing equations for arithmetic and geometric sequences. It will be referred to as Checkpoint 4A.

a. Write an explicit and recursive rule for $t(n) = 1, 4, 7, 10, \ldots$

b. Write an explicit and recursive rule for $t(n) = 3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \ldots$

In parts (c) and (d), write an explicit rule for the sequence given in the $n \rightarrow t(n)$ tables.

c. An arithmetic sequence

n	$t(n)$
0	
1	17
2	
3	3
4	

d. A geometric sequence

n	$t(n)$
0	
1	
2	7.2
3	8.64
4	

e. If an arithmetic sequence has $t(7) = 1056$ and $t(12) = 116$, what is $t(4)$?

Check your answers by referring to the Checkpoint 4A materials.

If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 4A materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do problems like these quickly and easily.

4-43. Wet World has an 18-foot-long water slide. The angle of elevation of the slide (the angle it forms with a horizontal line) is 50°. At the end of the slide, there is a 6-foot drop into a pool. After you climb the ladder to the top of the slide, how many feet above the water level are you? Draw a diagram. Help (Html5)⇔Help (Java)

4-44. Find the slope and y-intercept of each line below. Help (Html5)⇔Help (Java)

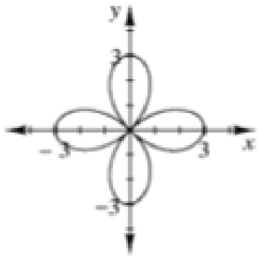
a. $y=-\frac{6}{5}x-7$

b. $3x - 2y = 10$

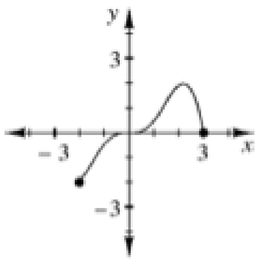
c. The line that goes through the points $(5, -2)$ and $(8, 4)$

4-45. Examine the graph of each relation below. For each part below, decide if the relation is a function and then state the domain and range. Help (Html5)⇔Help (Java)

a.



b.



4-46. Solve the system of equations below. Help (Html5)⇔Help (Java)

$$2^{(x+y)} = 16$$
$$2^{(2x+y)} = \frac{1}{8}$$