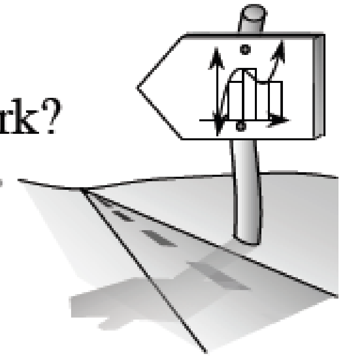
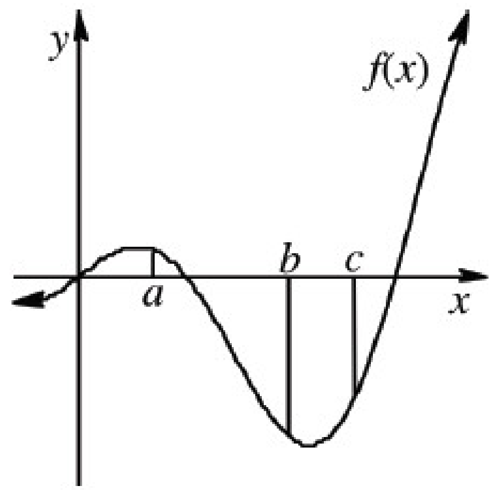
4.1.3 How do the limits of integration work?



Properties of Definite Integrals

4-27. PROPERTIES OF INTEGRALS
Consider the integrals below. For each integral, draw and shade the region for a generic function, $f(x)$. Simplify each expression integral and summarize each case on your paper.



a. $\int_a^a f(x)\,dx$

b. $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx$

c. $\int_b^a f(x)\,dx + \int_a^b f(x)\,dx$

d. $\int_b^a k \cdot f(x)\,dx$, where k is a constant

4-28. PROPERTIES OF INTEGRALS, CONTINUED
You have developed methods of simplifying integrals with a single function. What happens when we combine two functions? Investigate the following relationship:

$$\int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

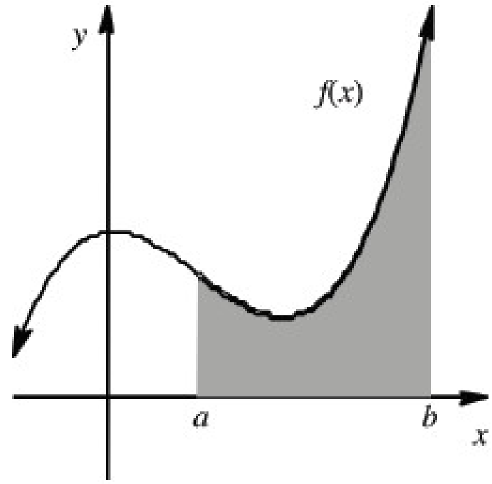
a. Find $\int_0^2 x\,dx + \int_0^2 3x\,dx$.

b. Find $\int_0^2 (4x)\,dx$

c. Rewrite the expression $\int_a^b f(x)\,dx + \int_a^b g(x)\,dxd$ into a simplified form.

4-29. TRANSLATIONS OF FUNCTIONS

Examine what happens to the area of a region when a function is translated. Some cases to consider are listed below, but do not feel restricted to them. When finished, summarize your findings clearly.



a. Does $\int_a^b [f(x)+k]\,dx = \int_a^b f(x)dx + k$? Explain why or why not.

b. Does $\int_a^b f(x)\,dx = \int_{a+c}^{b+c} f(x)\,dx$? Explain why or why not.

c. Does $\int_a^b f(x)\,dx = \int_{a+c}^{b+c} f(x-c)\,dx$? Explain why or why not.

d. Does $\int_a^b f(x)\,dx = \int_a^b f(x+c)\,dx$? Explain why or why not.

e. Summarize the integral equations that are correct on your paper.

4-30. In your team, write general formulas for all the properties of integrals you discovered today.

Review & Preview

4-31. Differentiate the following with respect to x. That is, find $\frac{dy}{dx}$. Help (Html5)⇔Help (Java)

a. $y = \frac{x+1}{x}$

b. $y = \cos x + \sin x$

c. $y = x \cdot \sqrt[3]{x^2}$

d. $y = (6 - 5x)(1 - 2x)$

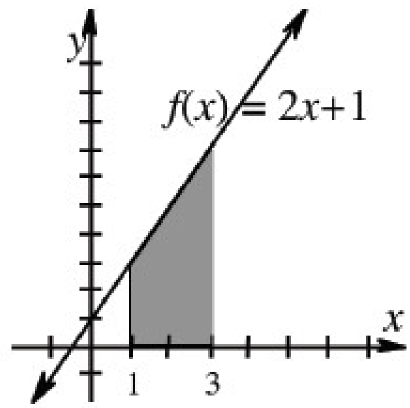
4-32. Evaluate the following integrals without a calculator. Then write a statement about the connection between them. Check your answer with a calculator. Help (Html5)⇔Help (Java)

a. $\int_2^9 8x\,dx$

b. $\int_2^9 (8x+5)\,dx$

c. $\int_2^9 5\,dx$

4-33. Given the graph to the right of $f(x) = 2x + 1$, find: Help (Html5)⇔Help (Java)



a. $\int_1^3 (2x+1)\,dx$

b. $\int_1^3 (2t+1)\,dt$

c. What is the difference between the expressions in parts (a) and (b)?

4-34. Complete the following. Help (Html5)⇔Help (Java)

a. Find the equations of the two lines tangent to the curve $f(x) = x^3 - x^2 + x + 1$ that have a slope of 2.

b. Determine the equations of the lines perpendicular to the tangent lines from part (a) at their points of tangency to the graph.

4-35. Given $f(x) = \sin x$, $g(x) = x^2$ and $h(x) = \frac{1}{x}$, use compositions to express each of the following functions. Help (Html5)⇔Help (Java)

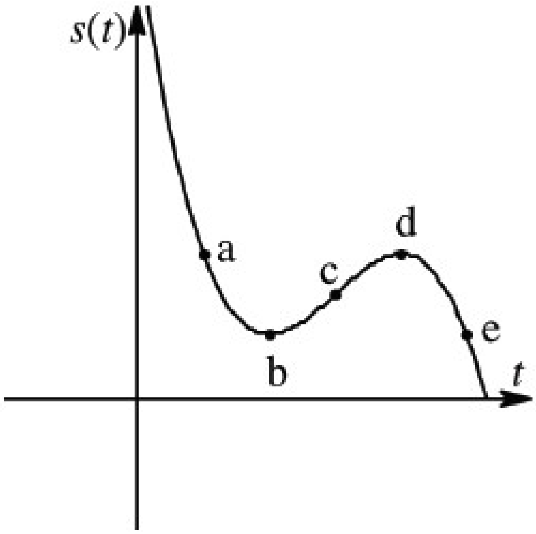
a. $y = \sin(x^2)$

b. $y = \sin^2(x)$

c. $y = \csc(x)$

d. $y = \csc^2\left(\frac{1}{x}\right)$

4-36. Using the distance vs. time graph below, determine if the velocity is positive, negative or zero at each point listed in the graph. Help (Html5)⇔Help (Java)



4-37. Sketch a graph of $f(x) = x^3 - 2x^2$. At what point(s) will the line tangent to $f(x)$ be parallel to the secant line through $(0, f(0))$ and $(2, f(2))$? Help (Html5)⇔Help (Java)

4-38. On your paper, sketch a graph of $f(x) = x^3 + 3x^2 - 45x + 8$. Help (Html5)⇔Help (Java)

a. Find the slope of the line tangent to the curve at $x = -2$.

b. Find the point on the curve where the slope is the smallest (steepest negative slope). What is the name of this point?

4-39. Given the function $f(x) = \begin{cases} 2x^2 - 4 & \text{for } x \le 3 \\ -2x - 5 & \text{for } x > 3 \end{cases}$ find: Help (Html5)⇔Help (Java)

a. $\lim\limits_{x \to 3^+} f(x)$

b. $\lim\limits_{x \to 3^-} f(x)$

c. What do your results above tell you about $f(x)$?