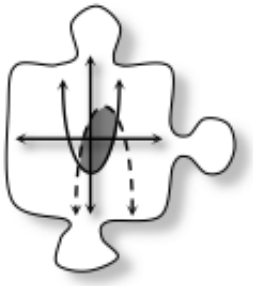
4.2.1 How can I solve inequalities?



Solving Inequalities with One or Two Variables

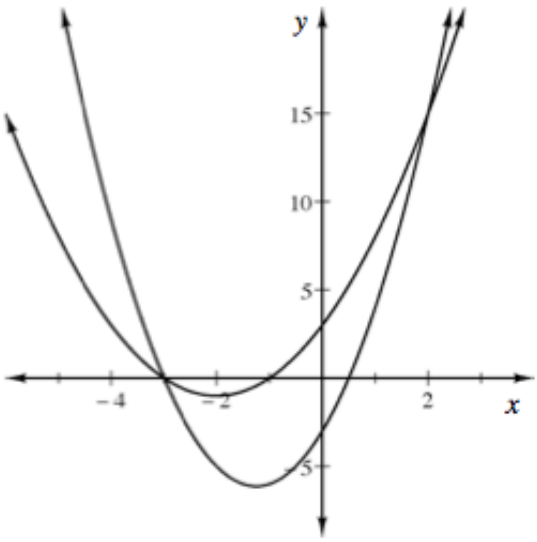
In this chapter, you developed many strategies for solving equations with one variable and systems of equations with two variables. But what if you want to solve an inequality or system of inequalities instead? Today you will explore how to use familiar strategies to find solutions for an inequality. As you work, the questions below can help focus team discussions:

What strategy should we use?

How can we know if this solution is correct?

How can we be sure we found all of the solutions?

4-58. In the previous section, you learned how to use the graph of a system to solve an equation. How can the graphs of $y = 2x^2 + 5x - 3$ and $y = x^2 + 4x + 3$ (shown at right) help you solve an *inequality*? Consider this as you answer the questions below. Explore the graph using 4-58 Student eTool (Desmos).



a. How are the solutions of $2x^2 + 5x - 3 = x^2 + 4x + 3$ represented on this graph? What are the solutions?

b. Obtain a Lesson 4.2.1 Resource Page from your teacher. On the resource page, label each graph with its equation and highlight each function with a different color. How did you decide which graph matches which function?

c. On the graph, identify the x-values for which $2x^2 + 5x - 3 \leq x^2 + 4x + 3$. How did you locate the solutions? How many solutions are there? Find a way to describe all of the solutions.

d. How can these solutions be represented on a number line? Locate the number line labeled with $2x^2 + 5x - 3 \leq x^2 + 4x + 3$ below the graph on your resource page. Use a colored marker to highlight the solutions to the inequality on the number line.

e. What about the inequality $2x^2 + 5x - 3 > x^2 + 4x + 3$? What are the solutions to this inequality? Represent your solutions algebraically and on a number line.

4-59. Consider the inequality $4|x+1|-2>6$.

a. How many boundary points are there? Remember that, in this case, a boundary point would be the smallest number that will make the inequality not true. What are the boundary points? Should they be marked with filled or unfilled circles? Make the appropriate markings on a number line.

b. Which portions of the number line contain the solutions for this inequality? How many regions do you need to test? Represent the solutions algebraically and on a number line.

4-60. Burt and Ernie were solving the inequality $2x^2 + 5x - 3 < x^2 + 4x + 3$. They were looking at the graph in problem 4-58 when Burt had an idea. "*Can't we change this into one parabola and solve our inequality that way*?" he said.

Ernie asked, "*What do you mean*?"

"*Can't we find the solutions by looking at the graph of* $f(x) = x^2 + x - 6$?" Burt replied.

a. Where did Burt get the equation $y = x^2 + x - 6$?

b. Try Burt's idea. Graph the parabola and show how it can be used to solve the original inequality.

c. "*Just a minute!*" mumbled Ernie, "*I think I have a short cut. Instead of graphing the parabola, can't we just rewrite the original inequality as* $x^2 + x - 6 < 0$ *and then solve the equation* $x^2 + x - 6 = 0$*? This would give us the boundary points and then we could test numbers to find the regions that contain the solutions*." Check Ernie's short cut. Does it give the same solution?

d. Use any method to solve the inequality $x^2 - 3x - 10 \geq 0$.

4-61. Next, Burt and Ernie were working on solving the inequality $4|x+1|-2>6$ from problem 4-59. This time, Ernie had an idea. "*Why don't we find the solutions to this by graphing a system of equations like we did in problem 4-58*?"

a. What system of equations should they graph?

b. Graph the system and explain how you can use it to find the solutions to $4|x+1|-2>6$.

4-62. In problem 4-58 you looked at solutions to an inequality with one variable (x). Now consider the system of inequalities with two variables (x and y) below.

$$y - 2x^2 + 5x - 3$$
$$y < x^2 + 4x + 3$$

a. Which points make both inequalities true? For example, does the point (−3, 0) make both inequalities true? What about (−1, 1)? (1, 5)? Refer back to yourLesson 4.2.1A Resource Page to help you think about these questions.

b. What is the difference between a solution to the *system* of inequalities above and a solution to the

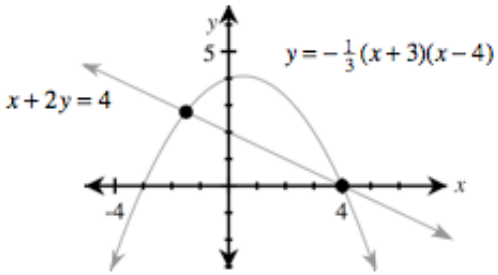
inequality found in problem 4-58?

c. How are the graphs of the equations $y = 2x^2 + 5x - 3$ and $y = x^2 + 4x + 3$ related to the graph of the system of inequalities?

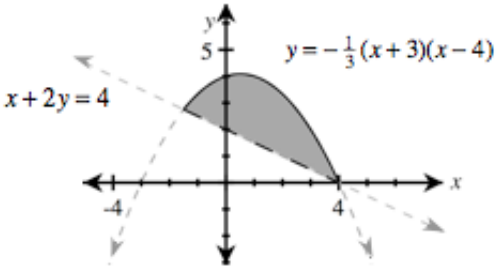
d. With your team, find a way to represent all of the solutions to the system of inequalities on the Lesson 4.2.1A Resource Page graph.

4-63. For each of the following graphs, find an equation, inequality, or system that could have the solution shown. Note that the equations for the line and the parabola are given. Explore the graph using 4-63 Student eTool (Desmos).

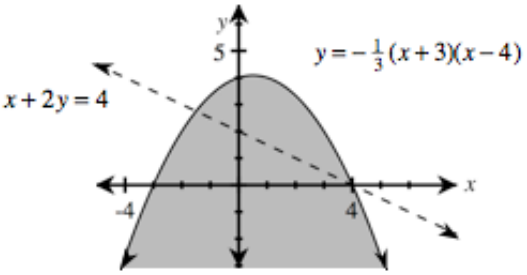
a.



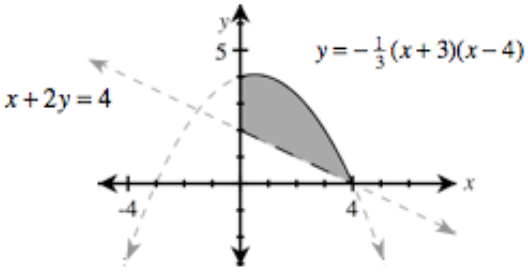
b.



c.



d.



4-64. LEARNING LOG

Now you will reflect for a third time about the meaning of solutions. What does the solution to an *inequality* or a *system of inequalities* mean? Does it matter if the inequality has one variable or two? Create a Learning Log entry that expands on your thinking about the

meaning of a solution. Title this entry "The Meaning of a Solution, Part 3" and label it with today's date.

4-65. Find boundary points for each of the following inequalities. Draw the boundaries on a number line and shade the solution regions. Help (Html5)⇔Help (Java)

a. $3x + 2 \geq x - 6$

b. $2x^2 - 5x < 12$

4-66. Solve the following inequalities and draw a number line graph to represent each solution. Help (Html5)⇔Help (Java)

a. $|2x+3| < 5$

b. $|2x+3| \geq 5$

c. $|2x-3| < 5$

d. $|2x-3| \geq 5$

e. $|3-2x| < 5$

f. $|3-2x| \geq 5$

g. Describe any relationships you see among these six problems.

4-67. Solve each equation for y so that it could be entered into a graphing calculator. Help (Html5)⇔Help (Java)

a. $5 - (y - 3) = 3x$

b. $4(x + y) = -2$

4-68. Solve each equation below. Remember to check for extraneous solutions. Help (Html5)⇔Help (Java)

a. $(y - 3)^2 = 2y - 10$

b. $|y-3| = 2y-10$

4-69. Add, subtract, multiply, or divide the following rational expressions. Then simplify your expression, if possible. Help (Html5)⇔Help (Java)

a. $\frac{x-4}{2x^2+9x-5} + \frac{x+3}{x^2+5x}$

b. $\frac{4x^2-11x+6}{2x^2-x-6} - \frac{x+2}{2x+3}$

c. $\frac{(x+4)(2x-1)(x-7)}{(x+8)(2x-1)(3x-4)} \div \frac{(4x-3)(x-7)}{(x+8)(3x-4)}$

d. $\frac{2m^2+7m-15}{m^2-16} \cdot \frac{m^2-6m+8}{2m^2-7m+6}$

4-70. Using the technique of completing the square, solve $x^2 + 12x + 15 = 75$ for x. Help (Html5)⇔Help (Java)

4-71. Factor each expression in parts (a) and (b). Then, in parts (c) and (d), factor and simplify each expression. Help (Html5)⇔Help (Java)

a. $bx + ax$

b. $x + ax$

c. $\frac{ax+a}{x^2+2x+1}$

d. $\frac{x^2-b^2}{ax+ab}$

4-72. Graph the four inequalities below on the same set of axes. Help (Html5)⇔Help (Java)

i. $2y \geq x - 3$ ii. $x - 2y \geq -7$

iii. $y \leq -2x + 6$ iv. $-9 \leq 2x + y$

a. What type of polygon is formed by the solution of this set of inequalities? Write a convincing argument to justify your answer.

b. Find the vertices of the polygon. If your graph is very accurately drawn you will be able to determine the points from the graph. If it is not, you will need to solve the systems (pairs) of equations that represent the corners of your graphs.

4-73. Solve the following absolute value inequalities. Help (Html5)⇔Help (Java)

a. $|x-4| < 9$

b. $\left|\frac{1}{2}x - 45\right| \geq 80$

c. $|2x-5| \leq 2$

4-74. Your family plans to buy a new air conditioner. They can buy the Super Cool X1400 for \$800, or they can buy the Efficient Energy X2000 for \$1200. Both models will cool your home equally well, but the Efficient Energy model is less expensive to operate. The Super Cool X1400 will cost \$60 per month to operate, while the Efficient Energy X2000 costs only \$40 per month to operate. Help (Html5)⇔Help (Java)

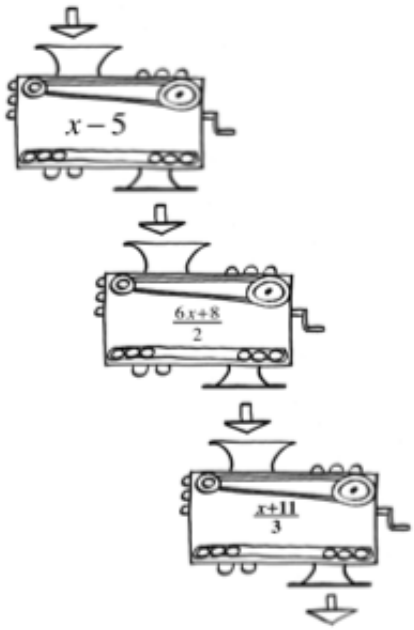
a. Write an equation to represent the cost of buying and operating the Super Cool X1400 where C = cost and m = months.

b. Write an equation to represent the cost of buying and operating the Efficient Energy X2000.

c. How many months would your family have to use the Efficient Energy model to compensate for the additional cost of the original purchase?

d. Figuring your family will only use the air conditioner for 4 months each year, how many years will you have to wait to start saving money overall?

4-75. MARVELOUS MARK's FUNCTION MACHINES
Mark has set up a series of three function machines that he claims will surprise you. Help (Html5)⇔Help (Java)



a. Try a few numbers. Are you surprised by your results?

b. Carrie claims that she was not surprised by her results. She also says that she can show why the sequence of machines does what it does by simply dropping in a variable and writing out step-by-step what happens inside each machine. Try it. (Use something like c or m.) Be sure to show all of the steps.

4-76. Multiply or divide the rational expressions below. Write each answer in simplified form. Help (Html5)⇔Help (Java)

a. $\frac{(x-3)^2}{2x-1} \cdot \frac{2x-1}{(3x-14)(x+6)} \cdot \frac{x+6}{x-3}$

b. $\frac{4x^2+5x-6}{3x^2+5x-2} \div \frac{4x^2+x-3}{6x^2-5x+1}$

4-77. Find all of the points at which the parabolas below intersect. Write your solution(s) in (x, y) form. Help (Html5)⇔Help (Java)

$$y = x^2 - x + 12$$
$$y = 2x^2 + 3x + 7$$

4-78. Find the equation (in $y = mx + b$ form) of each line described below. Help (Html5)⇔Help (Java)

a. A line with slope $\frac{1}{2}$ passing through the point (6, 1).

b. The line $y = 2x + b$ passing through the point (1, 4).