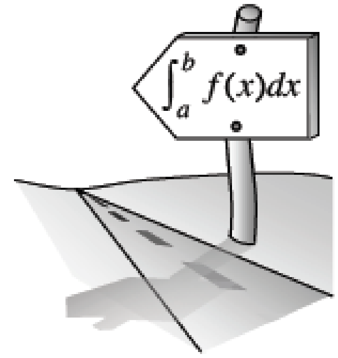
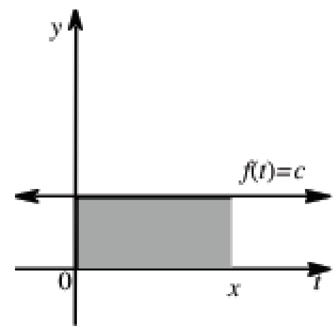
4.2.1 Does the lower bound matter?



Deriving "Area Functions"

4-40. Slope Functions



Finding slope functions is powerful because they offer the ability to express the rate of change of a function for all values x in the domain. However, what about the area under a function? How can we also write an "area function," A(x), that will calculate the area for all values t in the domain?

Find the area of the following regions:

a. $A(0) = \int_0^0 5\ dt$

b. $A(1) = \int_0^1 5\ dt$

c. $A(2) = \int_0^2 5\ dt$

d. $A(x) = \int_0^x 5\ dt$

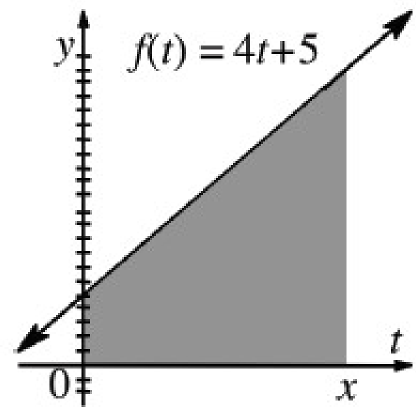
e. Generalize your findings for all constant functions. That is, write an expression for, $A(x) = \int_0^x c\ dt$ where c is a constant.

4-41. What if $f(t)$ is not a constant function? Examine the area function $A(x)$ of a line with slope $m \neq 0$.

a. Evaluate $\int_0^2 (4t+5)\ dt$.

b. Evaluate $\int_0^9 (4t+5)\ dt$.

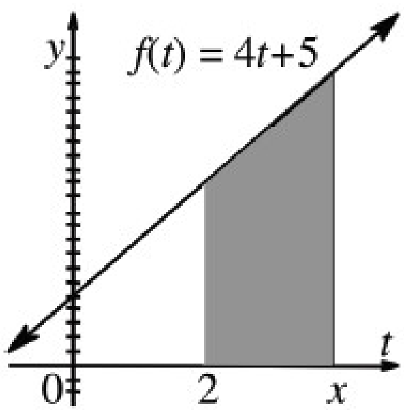
c. Find the area function $A(x) = \int_0^x (4t+5)\,dt$.



d. What does $\int_0^x (4t+5)\,dt$ compute?

e. Generalize $A(x) = \int_0^x (mt+b)\,dt$. What is the significance of $mt+b$?

4-42. Does it matter what the lower bound of the integral is? What if the lower bound is not 0, but instead is another constant?



a. Discuss with your study team how $A(x)$ changes as the lower bound changes. Test your conjecture by comparing $A(x)$ and $B(x)$ below. $A(x) = \int_0^x (4t+5)\,dt$

$B(x) = \int_2^x (4t+5)\,dt$

b. Using a method similar to that used in problem 4-29 part (c), find the area function $B(x)$. Then, compare it to $A(x)$.

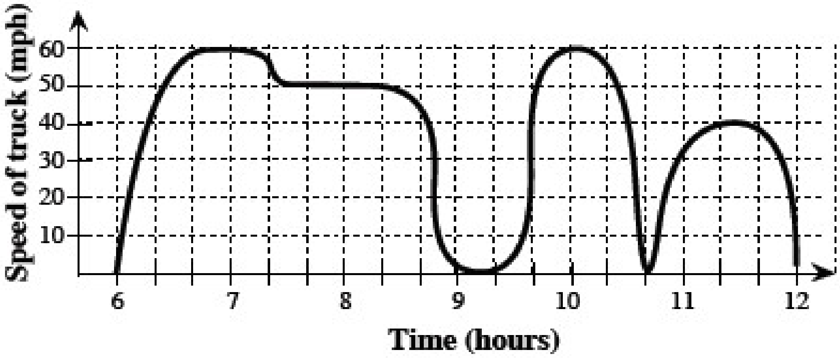
c. Demonstrate algebraically that . $B(x) = \int_2^x (4t+5)\,dt = A(x) - A(2)$ Also demonstrate this relationship geometrically using of area.

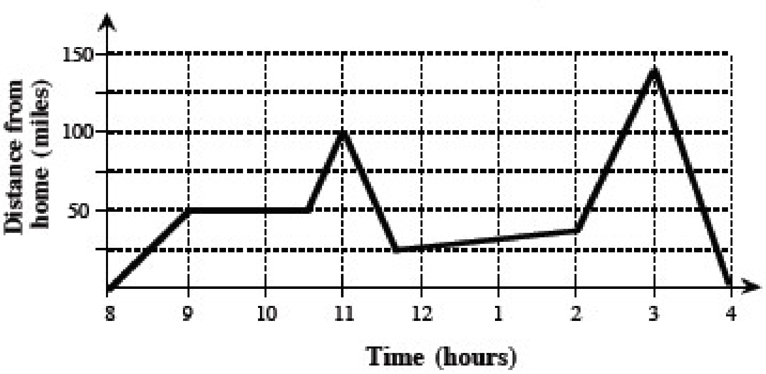
d. Do the two area functions grow at the same rate? Does $A'(x) = B'(x)$. Is this true? Why or why not?

e. Create an expression using $A(x)$ to evaluate $G(x) = \int_c^x (4t+5)\,dt$. Explain geometrically what $G(x)$ measures.

Review & Preview

4-43. Review your results from the Freeway Fatalities problem in Chapter 1. Write a complete statement describing the relationship between the distance of the truck from home and its velocity. How can we determine the distance traveled from the velocity vs. time graph? How can we determine the velocity from the distance vs. time graph? Help (Html5)⇔Help (Java)





4-44. Find the general antiderivative, $F(x)$, for each function, $f(x)$, below. Help (Html5)⇔Help (Java)

a. $f(x) = \cos x$

b. $f(x) = -\frac{2}{x^2}$

c. $f(x) = -9x^{1/3}$

4-45. Use a Riemann sum with 20 rectangles to approximate the following integrals. Then use the numerical integration feature of your graphing calculator to check your answer. Help (Html5)⇔Help (Java)

a. $\int_0^4 (2 - 4x^{3/2})$

b. $\int_1^8 \sqrt{4x+3}\, dx$

4-46. Explain why there are an infinite number of antiderivatives for each function. Demonstrate this fact with an example. Help (Html5)⇔Help (Java)

4-47. Use the Power Rule to find $f'(x)$ for $f(x) = (x^2 + 1)(x - 4)$ by first expanding $f(x)$. Then, find the equation of the line tangent to $f(x)$ at $x = -3$. Help (Html5)⇔Help (Java)

4-48. If $h(x) = f(x) \cdot g(x)$, then does $h'(x) = f'(x) \cdot g'(x)$? Test this idea on $f(x) = (x^2 + 1)(x - 4)$ using your results from problem 4-47. Thoroughly record your results. Help (Html5)⇔Help (Java)

4-49. Given $h(x)$ below, define functions $f(x)$ and $g(x)$ so that $h(x) = f(g(x))$. (Note: $f(x) \neq x$ and $g(x) \neq x$) Help (Html5)⇔Help (Java)

a. $h(x) = \sqrt{\sin(x^2) + 1}$

b. $h(x) = (3x^3 - 12)^2 + 2$

4-50. If n is a positive integer write an integral to represent $\lim\limits_{n\to\infty} \frac{1}{n}\left[\frac{1}{\left(\frac{1}{n}\right)} + \frac{1}{\left(\frac{2}{n}\right)} + \ldots + \frac{1}{\left(\frac{n}{n}\right)}\right]$. Help (Html5)⇔Help (Java)

4-51. What is the equation of the vertical line that will divide $\int_0^6 5\, dx$ in half? Is this the same line that will divide $\int_0^6 5x dx$ in half? Help (Html5)⇔Help (Java)