4.2.2 How can I organize the possibilities?

Using Systems to Solve a Problem

Businesses and industries often use equations and inequalities to model their services and production. Creating a system of equations and inequalities allows them to mathematically optimize their operation and maximize profits. Today you will investigate this technique.

4-79. THE TOY FACTORY

Otto Toyom builds toy cars and trucks. To make each car, he needs 4 wheels, 2 seats, and 1 gas tank. To make each truck, he needs 6 wheels, 1 seat, and 3 gas tanks. His storeroom has 36 wheels, 14 seats, and 15 gas tanks. He is trying to decide how many cars and trucks to build so he can make the largest possible amount of money when he sells them. Help Otto figure out what his options are. What are all of the choices he could make about how many cars and how many trucks he will build? Make a list of all possible combinations. Then plot the number of possible cars and trucks in the first quadrant of a graph.

4-80. Otto wants to make as much profit as possible. Use your list from problem 4-79 to find which combination of cars and trucks will make the most profit based on the information below.

a. Which of Otto's options gives him the greatest profit if he makes $1 on each car and $1 on each truck he sells? How do you know?

b. The market has changed, and Otto can now make $2 for each truck but only $1 for each car. What is his best choice for the number of cars and the number of trucks to make in this situation? How can you be sure? Explain.

4-81. To convince Otto that your recommendation was a good one, you probably had to show many calculations in problem 4-80. Now, you will take another look at Otto’s business using algebra and graphing tools.

a. Write three inequalities to represent the relationship between the number of cars (x), the number of trucks (y), and the number of:

    i. wheels

    ii. seats

iii. gas tanks

b. Graph this system of inequalities on the same set of axes you used for problem 4-79. Shade the solution region lightly. Why is it okay to assume that $x \geq 0$ and $y \geq 0$?

c. What are the vertices of the pentagon that outlines your region? Explain how you could find the exact coordinates of those points if you could not read them easily from the graph.

d. Are there any points in the solution region that represent choices that seem more likely to give Otto the maximum profit? Where are they? Why do you think they show the best choices?

e. Write an equation to represent Otto's total profit (P) if he makes \$1 on each car and \$2 on each truck. What if Otto ended up with a profit of only \$8? Show how to use the graph of the profit equation when $P = 8$ to figure out how many cars and trucks he made.

f. Which points do you need to test in the profit equation to get the maximum profit? Is it necessary to try all of the points? Why or why not?

g. What if Otto got greedy and wanted to make a profit of \$14? How could you use a profit line to show Otto that this would be impossible based on his current pricing?

4-82. Find Otto's highest possible profit if he gets \$3 per car and \$2 per truck. Find the profit expression and find the best combinations of cars and trucks to maximize the profit.

METHODS AND MEANINGS

MATH NOTES

Inequalities with Absolute Value

If k is any positive number, an inequality of the form $|f(x)| > k$ is equivalent to the statement $f(x) > k$ OR $f(x) < -k$.

For example, $|2x-17| > 9$ is equivalent to $2x - 17 > 9$ or $2x - 17 < -9$. Solving yields $x > 13$ or $x < 4$.

$|f(x)| < k$ is equivalent to the statement $-k < f(x) < k$. Another way to write this is $f(x) > -k$ AND $f(x) < k$. For example, $|x+4| < 9$ is equivalent to $-9 < x + 4 < 9$. Solving yields $-13 < x < 5$, that is, $x > -13$ and $x < 5$.

4-83. Solve the system of equations below. What sub-problems did you need to solve? Help (Html5)⇔Help (Java)

$$x + 2y = 4$$
$$2x - y = -7$$
$$x + y + z = -4$$

4-84. Solve each of the following inequalities. Express the solutions algebraically and on a number line. Help (Html5)⇔Help (Java)

a. $3x - 5 \leq 7$

b. $x^2 + 6 > 42$

4-85. Three red rods are 2 cm longer than two blue rods. Three blue rods are 2 cm longer than four red rods. How long is each rod? Help (Html5)⇔Help (Java)

4-86. Simone has been absent and does not know the difference between the graph of $y \leq 2x - 2$ and the graph of $y < 2x - 2$. Explain thoroughly so that she completely understands what points are excluded from the second graph and why. Help (Html5)⇔Help (Java)

4-87. This problem is a checkpoint for solving for one variable in an equation with two or more variables. It will be referred to as Checkpoint 4B.

Rewrite the following equations so that you could enter them into a graphing calculator. In other words, solve for y.

a. $x - 3(y + 2) = 6$

b. $\frac{6x-1}{y} - 3 = 2$

c. $\sqrt{y-4} = x+1$

d. $\sqrt{y+4} = x+2$

Check your answers by referring to the Checkpoint 4B materials.

If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 4B materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do problems like these quickly and easily.

4-88. Think about the axis system in the two-dimensional coordinate plane. What is the equation of the x-axis?

What is the equation of the y-axis? Help (Html5)⇔Help (Java)

4-89. Sammy has a 10-foot wooden ladder, which he needs to climb to reach the roof of his house. The roof is 12 feet above the ground. The base of the ladder must be at least 1.5 feet from the base of the house. How far is it from the top step of the ladder to the edge of the roof? Draw a sketch. Help (Html5)⇔Help (Java)