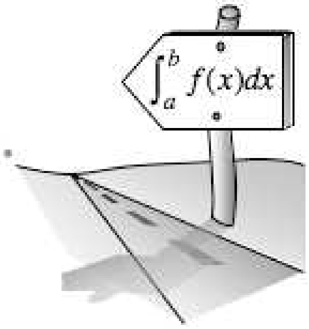
4.2.2 What is the meaning of + C?

Indefinite and Definite Integrals



4-52. Tommy is trying to find the area $\int_2^3 f(t)\,dt$. He already knows that: $\int_0^x f(t)\,dt = 9x^2 - 2$.

$\int_2^3 f(t)\,dt$

Help him find .

MATH NOTES

Area Functions

Part (c) of problem 4-42 showed us that if $A(x)$ is the function that determines the area under a curve between the fixed point 0 and the variable point x, then:

$$\int_c^x f(t)\,dt = A(x) - A(c)$$

Since $A(c)$ is a constant, the area function can be rewritten as

$$\int_c^x f(t)dt = A(x) - A(c) = A(x) + C$$

for some constant C. Since the lower bound is arbitrary, this shows that *any* two area functions of $f(x)$ differ by a constant. We use the symbol $\int f(x)\,dx$ to write this *general* area function. In other words,

$$\int f(x)dx = A(x) + C$$

We call $\int f(t)\ dt$ an **indefinite integral**because we do not specify fixed bounds and are instead seeking a *general* area function. Note that $\int f(t)\ dt$ and $\int f(x)\,dx$ only differ by the choice of dummy variable.

A **definite integral** is an integral for which bounds are defined. Although they are related, there are some critical differences. The most important difference is that an indefinite integral is *a function* while a definite integral is a *number*. See the examples below:

$$\int (2x+3)dx \rightarrow \textbf{indefinite integral} \qquad \int_2^9 (2x+3)\,dx \rightarrow \textbf{definite integral}$$

4-53. Use the Math Note above to answer the following questions.

a. Which type of integral would you use to find the area under $f(x) = 3x^5 - 8x + 2$ from $x = -4$ to$x = 6$?

b. Which type of integral would find all the functions whose slope function is $f(x) = 3\tan x$?

c. Which type of integral leaves the answer in a numerical form?

d. Which type of integral results in a function?

e. Which integral has infinitely many solutions?

4-54. Sketch the region represented by the integral below. Then use geometry to find general area functions for each integral below.

a. $\int 2\,dx$

b. $\int (5x+2)\,dx$

c. $\int \left(\frac{3}{2}-k\right)dk$

4-55. Use your results from problem 4-54 and the formula to evaluate the following areas.

a. $\int_{3}^{8} 2\,dx$

b. $\int_{-2}^{5} 8\,dx$

c. $3\int_{2}^{6} (5x+2)\,dx$

d. $\int_{1}^{3} 5x\,dx+\int_{1}^{3} 2\,dx$

e. $\int_{4}^{10} \left(\frac{3}{2}-k\right)dk$

f. $\int_{0}^{-2} \left(\frac{3}{2}-k\right)dk$

Review & Preview

4-56. Ji Hee is trying to find the area $\int_{-1}^{5} g(m)\,dm$. She already knows that: Homework Help ✎

$\int_{0}^{x} g(m)dm=\frac{4x+1}{x+2}$.

Help her find:

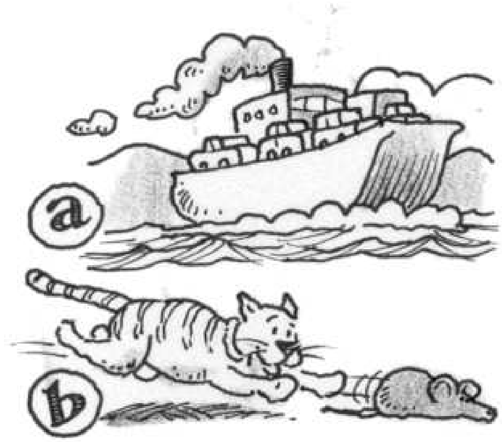
a. $2\int_{0}^{3} g(m)dm$

b. $\int_{-1}^{0} g(m)dm$

c. $\int_{-1}^{5} g(m)\,dm$

4-57. Mateo and Ignacio want to calculate $\int_{-2}^{-5} f(x)\,dx$ of an even function $f(x)$. They know $\int_{2}^{5} f(x)\,dx = -3$. Mateo thinks the answer will be −3 while Ignacio thinks the answer will be 3. Who is correct? Explain. Homework Help ✎

4-58. Compare how distance and velocity are related with these two scenarios: Homework Help ✎

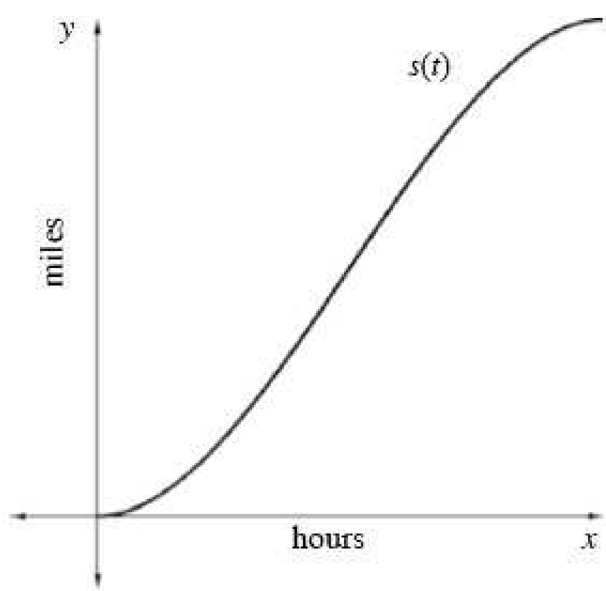


a. A ferry crosses the bay so that its distance (in miles) from the dock at time t is $d(t) = 1 - \cos t$. Find the velocity, $d'(t)$, at times $t = 1$, π, and 5 hours. Explain what concepts of calculus you applied in order to solve this problem.

b. When a cat chases a mouse, the cat's velocity, measured in feet per second, is $v(t) = 3t$. Sketch a graph and find the distance the cat ran in the first 5 seconds. Explain what concepts of calculus you applied in order to solve this problem.

c. Both (a) and (b) involve distance and velocity. However, each required a different method or approach. Describe the relationship between distance and velocity, mentioning the derivative and area under a curve.

4-59. While driving to work, Camille decided to keep track of the time and the distance she traveled. Taking the data that she gathered, she found the function $s(t)$ represented her distance as a function of time: $s(t) = -100t^3 + 150t^2$ Homework Help ✎



a. If Camille's trip took 1 hour, how far did she drive?

b. What was her average velocity?

c. Use your graphing calculator to find her maximum velocity. How far into the trip did she reach this speed?

4-60. Change the following limit of a Riemann sum for $A(f, -2 \le x \le 1)$ into an integral expression. Then, evaluate with your graphing calculator. Homework Help ✎

$$\lim_{n\to\infty}\sum_{i=1}^{n}\Delta x_i((x_i)^{1/3}-4)$$

4-61. If a function is differentiable at $x = c$, does that guarantee that $f(c)$ exists? Explain why or why not. Homework Help ✎

4-62. Find the equation of the tangent line to $f(x) = \sin x$ at $\frac{\pi}{3}$. Homework Help ✎

4-63. Differentiate. Homework Help ✎

a. $\frac{d}{dx}(7\cdot\sqrt[3]{x})$

b. $\frac{d}{dm}(3m^{-7}-7m^{3})$

c. $\frac{d}{dk}(k^{0})$

d. $\frac{d}{dt}((3t)(2t+5))$

4-64. Find the general antiderivative, $F(x)$, of the following functions: Homework Help ✎

a. $f(x) = 15x^4 + 4x - 3$

b. $f(x) = -2\cos x$

c. $f(x) = -4x^3 + 10$