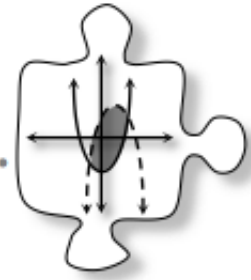# 4.2.3 How can I find the best combination?

Application of Systems of Linear Inequalities

The process of using linear systems to find the optimal solution to a problem with multiple constraints is called **linear programming**. You used this process while solving "The Toy Factory." Now you will work on a problem using this technique, only this time you can use a system of inequalities and will not need to list all of the possible outcomes.

4-90. SANDY DANDY DUNE BUGGIES

Jacklyn Toyom, CEO of the Sandy Dandy Dune Buggy Company and Otto's sister, has discovered that your team has found a way to optimize the profit for the Toy Factory. She would like to hire your team to help her company. Here is her letter:

Dear Study Team,

I was so impressed to hear about how you helped Otto maximize his profits at his Toy Factory! I think your team could help my company as well.

Here at the Sandy Dandy Dune Buggy Company we make two popular models of off-road vehicles: the Crawler and the Rover. Each week, we receive enough parts to build at most 15 Crawlers and 12 Rovers. The only exceptions to the supply of parts are the colored night lamps and high-definition speakers, which have to be specially manufactured for our off-road vehicles. Each of the Crawlers requires 5 of the lamps and 2 of the speakers. The Rover requires 3 lamps and 6 speakers. Our supplier is a small company and can only manufacture 81 of the lamps and 78 of the speakers for us each week.

Since we are also a small company, we have only 12 employees. By contract, the maximum number of hours each employee can work is 37.5 hours per week. It takes our employees 20 hours to assemble one Crawler and 30 hours to assemble one Rover.

Each Crawler sold brings in a profit of $500. The Rover, that is less expensive to manufacture than the Crawler, is very popular and sells for a profit of $1000 each.

I need a detailed proposal of how to maximize our profit that I can submit to our Board of Trustees. I look forward to a profitable business relationship!

Sincerely,
Ms. Jacklyn Toyom
CEO, Sandy Dandy Dune Buggy Company

Your Task: Find the best combination of Crawlers and Rovers to produce each week to maximize the company's profit. Create a detailed proposal to submit to Ms. Toyom that includes:

- The number of Crawlers and Rovers to manufacture each week.

- The maximum profit the company can expect to make.

- Calculations and graphs to justify your recommendation.

Constraints to keep in mind are the number of:

1. speakers available

2. lamps available

3. total employee hours each week

Discussion Points

How does this problem compare to "The Toy Factory" from the previous lesson?

What is the maximum number of hours for all of the employees that can be worked in one week?

How can we **justify** that we have found the most profitable combination of each vehicle to manufacture?

Further Guidance

4-91. After emailing a few questions to Ms. Toyom, your team received the following email:

From: "Ms. Toyom" <toyom@welovemath.com>
To: <studyteam@thinkingisgood.net>
Subject: Clarifications to your Questions

Dear Study Team,

Thank you for your questions. I am happy to clarify them. Our Board of Trustees requires the following information in your proposal:

1. A list of all of the constraints (to make sure you took them into consideration).

2. An inequality for each of the constraints.

3. A full-page graph showing all inequalities and the resulting solution region (use a different color for each inequality).

4. Calculations for each of the vertices on your solution region. List these points at their vertex.

5. Profit calculations, with maximum profit included on your graph.

Please make sure to include a cover letter summarizing your proposal. Also include a brief explanation for each of the items listed above.

Sincerely, Ms. Toyom

Further Guidance section ends here.

METHODS AND MEANINGS
MATH NOTES

Graphing Inequalities with Two Variables

To graph an inequality with two variables, first graph the boundary line or curve. If the inequality does not include equality (that is, if it is > or < rather than ≥ or ≤), then the graph of the boundary is dashed to indicate that it is not included in the solution. Otherwise, the boundary is a solid line or curve.

Once the boundary is graphed, choose a point that does not lie on the boundary to test in the inequality. If that point makes the inequality true, then the entire region in which that point lies is a solution. If that point makes the inequality false, then the entire region in which the point lies is not a solution. Examine the two examples below.
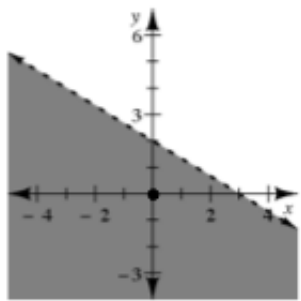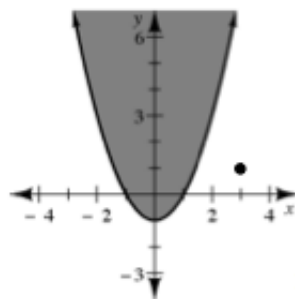
Test $(0,0)$:

$0 \overset{?}{<} -\frac{2}{3}(0)+2$

$0 \overset{?}{<} 2$

True, so shade below the line.

$y < -\frac{2}{3}x+2$

Test $(3,1)$:

$1 \overset{?}{\geq} 3^2-1$

$1 \overset{?}{\geq} 8$

False, so shade the region that does not contain the test point, that is, shade above the parabola.

$y \geq x^2-1$

Review & Preview

4-92. Solve the system of equations below algebraically and explain what the solution tells you about the graphs of the two equations. Help (Html5)⇔Help (Java)

$$3x + 2 = y$$
$$-9x + 3y = 11$$

4-93. Draw the graph of the system of inequalities below. Help (Html5)⇔Help (Java)

$$y \geq |x|-3$$
$$y \leq -|x|+5$$

   a. What polygon does the intersection form? Justify your answer.

   b. What are its vertices?

   c. Find the area of the intersection.

4-94. Solve each of the following inequalities. Express the solutions algebraically and on a number line. Help (Html5)⇔Help (Java)

   a. $3(x + 2) > 4x - 7$

   b. $3x^2 - 4x + 2 \leq x^2 + x + 6$

4-95. Solve the equations below. Help (Html5)⇔Help (Java)

   a. $\sqrt{x+15} = 5 + \sqrt{x}$

   b. $(y - 6)^2 + 10 = 3y$

4-96. Solve the system of equations below.

$$x + 3y = 16$$
$$x - 2y = 31$$

a. Now rewrite the system and replace x with x^2.

b. What effect will this have on the solution to the system? Solve the new system.

4-97. A line intersects the graph of $y = x^2$ twice. One point has an x-coordinate of -4, and the other point has an x-coordinate of 2.

a. Draw a sketch of both graphs, and find the equation of the line.

b. Find the measure of the angle that the line makes with the x-axis.