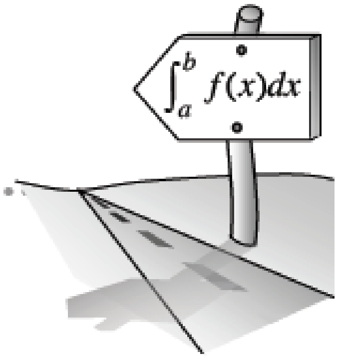
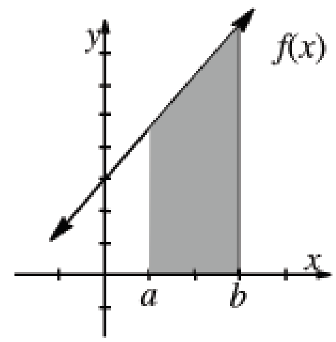
4.2.4 Proving and applying the FTC.



The Fundamental Theorem of Calculus

4-81. Write down what you know about the Fundamental Theorem of Calculus.



4-82. One part of the Fundamental Theorem states that $\int_a^b f(x)\,dx = F(b) - F(a)$. However, since this integral represents *area*, examine how $F(b)$ and $F(a)$ are also related to area.

a. If $F(x) = \int_0^x (2x+3)dx$ represents the area under the curve from 0 to x, then what do $F(a)$ and $F(b)$ represent?

b. Use the areas described in part (a) to show that $\int_a^b f(x)\,dx = F(b) - F(a)$.

c. Find $F'(t)$ given $F(x) = \int_0^x (2x+3)dx$.

4-83. From Lesson 4.2.3 you should have noticed a close relationship between derivatives and integrals just like with velocity and distance. In particular, we have seen that if we know the velocity, $v(t)$, at time t we can compute the distance traveled from time t = 0 to time x as $s(x) = \int_0^x v(t)\,dt$.

a. If the velocity function is $v(t) = 2t + 5$ in miles per hour, how far has the car traveled after 2 hours, after 5 hours, and after x hours?

b. How far did the car travel between 2 and 4 hours?

c. In the previous chapter we considered the instantaneous rate of change (derivative) of any function. Explain why you expect the derivative, $s'(t)$, of a distance function of a car to be the velocity, $v(t)$, of the car at that time.

4-84. Use the Fundamental Theorem of Calculus to evaluate each expression and compare the results.

a. $\frac{d}{dx}\int_3^x (3x-5)\,dx$

b. $\int_3^x \frac{d}{dx}(3x-5)\,dx$

c. $\frac{d}{dx}\int_3^5 (3x-5)\,dx$

d. $\int \frac{d}{dx}(3x-5)\,dx$

e. $\frac{d}{dx}\int (\cos(x^2))\,dx$

f. $\int_1^4 \frac{d}{dx}(\cos(x^2))\,dx$

4-85. Evaluate the following definite integrals by applying the Fundamental Theorem of Calculus. Test your solution by using the integration function on your calculator.

a. $\int_1^8 (2x^{-3})\ dx$

b. $\int_{\pi/4}^{\pi/2} (\sin x)\,dx$

c. $\int_4^9 \left(3\sqrt{x}\right)dx$

d. $\int_0^2 (3x^2-6x+2)\,dx$

e. $2\int_1^3 \frac{3}{x^2}\,dx$

f. $\int_{-1}^8 x^{1/3}\,dx$

g. $\int_3^9 (2x-20)\,dx + 2\int_{-2}^3 (x-10)\,dx$

h. $\int_0^{27} (9m^{-2/3}-2m^{-3})\,dm$

Review & Preview

4-86. Chang Young was attempting to evaluate the following area: $\int_1^5 (5x+2)\,dx$.

He showed the following steps:

$$\tfrac{5}{2}5^2+2(5)+C-\tfrac{5}{2}1^2+2(1)+C$$
$$\tfrac{5}{2}\cdot 25+10+C-\tfrac{5}{2}+2+C$$
$$\tfrac{125}{2}-\tfrac{5}{2}+12+2C$$
$$\tfrac{120}{2}+12+2C$$
$$60+12+2C$$
$$72+2C$$

He knows that this is a definite integral and there should not be any C's. Also, the teacher said the answer was 68. He needs your help to find his error and find out how to eliminate his $+\,2C$. Help (Html5)⇔Help (Java)

4-87. Evaluate the following integrals. Help (Html5)⇔Help (Java)

a. $\int (6x^3-2x+5)\,dx$

b. $\int_2^4 (6x^3-2x+5)\,dx$

c. $\int (9t^2-1)\,dt$

d. $\int_{-2}^2 (9t^2-1)\,dt$

e. $\int \left(\sin m+\tfrac{1}{3}m^2\right)dm$

f. $\int_{-\pi}^{\pi} \left(\sin m+\tfrac{1}{3}m^2\right)dm$

4-88. Rewrite the following integral expressions as single integrals. Help (Html5)⇔Help (Java)

a. $\int_{-3}^{-5} f(x)\,dx+\int_{-5}^{-3} g(x)\,dx$

b. $3\int_1^6 f(x)\,dx+5\int_1^6 g(x)\,dx$

c. $\int_6^{11} f(x)\,dx+\int_{11}^6 f(x)\,dx$

d. $\int_7^{10} f(t)\,dt - \int_7^9 f(t)\,dt$

4-89. Review Hanah's method for setting up a derivative. Use Hanah's definition of a derivative to differentiate $f(x) = 4x^2 - 9x - 1$. Help (Html5)⇔Help (Java)

4-90. Differentiate. Help (Html5)⇔Help (Java)

a. $\frac{d}{dx}(\sin(x-5))$

b. $\frac{d}{dx}(x^{82}-6)$

c. $\frac{d}{dm}(-2m^{3/2})$

d. $\frac{d}{dx}((x+6)^3)$

4-91. *Without your calculator*, describe the graph of $f(x) = x^3 + 12x^2 + 36x - 6$. A complete answer states where $f(x)$ is increasing, decreasing, concave up, concave down, and points of inflection. Help (Html5)⇔Help (Java)

4-92. Find the equation of the line tangent to $f(x) = x^3 + 12x^2 + 36x - 6$ at its point of inflection. Help (Html5)⇔Help (Java)

4-93. In Chapter 4, it was discovered that $f(x) = \sqrt[3]{x}$ was not differentiable at $x = 0$. Help (Html5)⇔Help (Java)

a. Why does the derivative of $f(x) = \sqrt[3]{x}$ not exist at $x = 0$?

b. Is $f(x) = \sqrt[3]{x^2}$ differentiable at $x = 0$? Why or why not?

c. What about $f(x) = \sqrt[3]{x^3}$?

d. Explain why there is a point of inflection at $x = 0$ for $f(x) = \sqrt[3]{x}$.

4-94. If $\int_2^4 g(x)\,dx = 6$, find $\int_2^2 (g(x)+3)\,dx$ and $\int_4^2 (3-5g(x))\,dx$