4-95. You will be working on the following problem over the next few days. Evidence will be presented by the prosecution that your team will need to verify or refute. Be careful with your analysis; the judge is less than patient with inaccuracies in arguments!

FAST TIMES

The Case of the Heavy-Footed Teacher

Your calculus teacher is in major trouble with the police department from a remote county. The police have issued tickets with very large fines for traveling at excessive speeds during a return trip from the mountains. In addition, if the tickets are upheld, your teacher's license will be suspended and insurance canceled! (Driving is one of the ways your teacher releases stress from teaching and without this release, upcoming tests are sure to become unbearably difficult.) The prosecution has gathered extensive data to support their case. Your task is to defend your teacher against these accusations.

FAST TIMES CAST OF CHARACTERS:

Judge Ironhand: Traffic court judge for 23 years. Has a reputation of being hard on sloppy evidence. Holds the record for fines levied on contempt charges.

Officer Tightman: The officer that issued the two tickets. Known to reach the monthly ticket quota during the first week.

Inspector Knoclew: In charge of analyzing traffic data for the police department.

Jerry Verigreen: The assistant District Attorney who is prosecuting the case. He is trying to impress his boss that he can be tough on crime. This is his first case.

Patrol Officer Rongway: Highway Patrol Pilot who is in charge of identifying speeders between the towns of Acelerton and Geprime.

The Defendant: Your teacher. Known for high speeds and hard tests. In desperate need of your help.

Council for the Defense: That's you!

4-96. FAST TIMES, Part One - The Prosecution Presents His Case

Prosecution: "Judge Ironhand, through extensive data collection we have solid evidence that the defendant was using excessive speeds while traveling through our tranquil law-abiding community. Through the outstanding work of our police department, we are able to bring justice to the defendant who clearly has no respect for our laws.

As my first witness I would like to call the officer who issued the traffic citations.

Officer Tightman, what evidence do you have that first suggested the defendant was using excessive speed?"
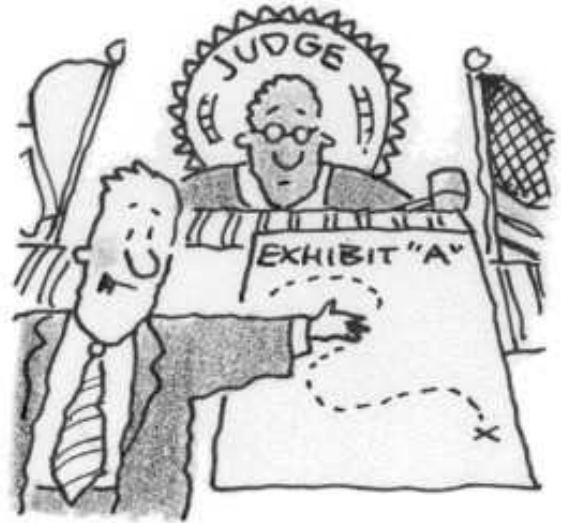
Officer Tightman: "The defendant claims to have departed Acelerton at 3:20 p.m. A receipt from the service station in Geprime shows that the defendant arrived at 5:38 p.m.

Even with our liberal laws allowing speeds of 65 mph, this is clearly too short of time to travel safely the distance shown below in miles."

Prosecution Exhibit A
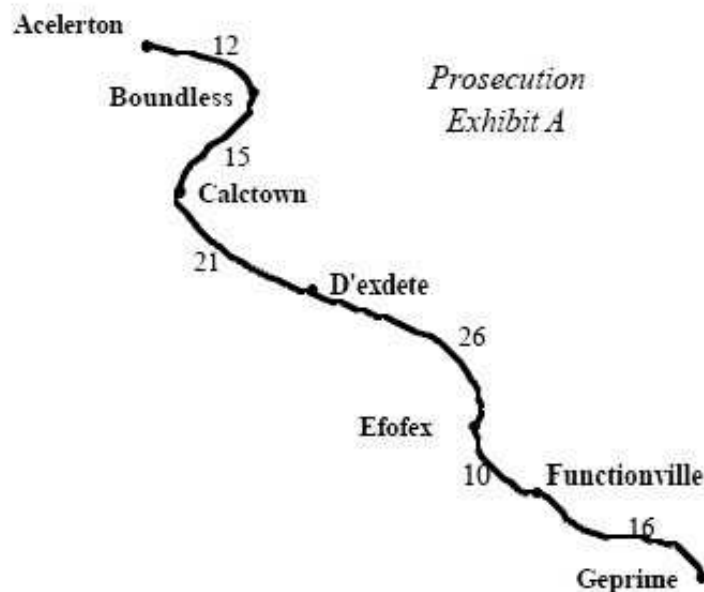
Judge Ironhand: "Council for the defense, how do you respond?"

Your task: Respond to the Judge. Is Officer Tightman correct in his claim? Justify your answer. Does this prove your teacher's guilt or innocence?

4-97. FAST TIMES, Part Two - Compelling Evidence

Prosecution: "Although I will acknowledge the mathematical prowess of my esteemed colleague, we have further evidence to suggest the defendant was traveling at an excessive speed in multiple instances.

Using the information from our roadside cameras, we have found the following function $s(t)$ for the distance (in miles) traveled by the defendant between Acelerton and D'exdete. I would like to now present *Exhibit B*, shown below, to demonstrate the first infraction." (Note: t is measured in hours.)

Prosecution Exhibit B
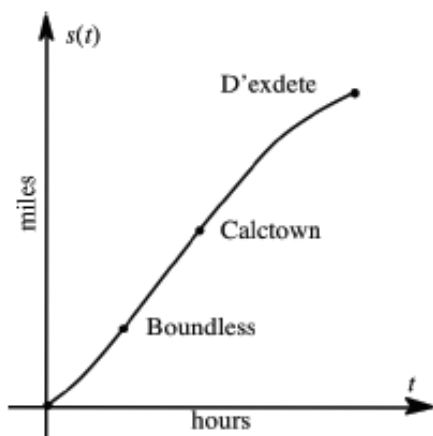
$$s(t) = -93.75t^3 + 93.75t^2 + 45t$$

Your task: Confirm or refute the prosecution's claim by finding the maximum velocity in the interval.

───── Review & Preview ─────

4-98. Examine the following integrals. Consider the multiple tools available for evaluating integrals and

use the best strategy for each. After evaluating the integral, write a short description of your method.
  Homework Help ✎

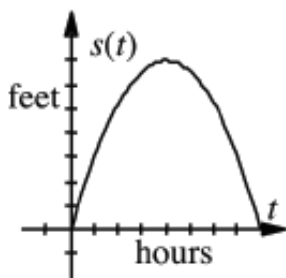a. $\displaystyle\int_{-\pi}^{\pi} \cos x\, dx$

b. $\displaystyle\int (5\sqrt{y} - \sin y)\, dy$

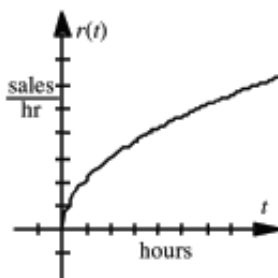c. $\displaystyle\int_{1}^{2.7183} \frac{1}{x}\, dx$

d. $\displaystyle\int (\sin^2 x + \cos^2 x)\, dx$
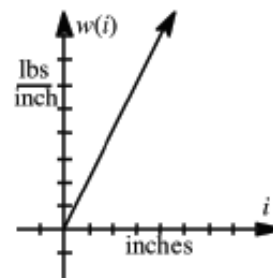
e. $\displaystyle\int \frac{e^x}{x}\, dx$

4-99. Using complete sentences, describe what each of the following graphs represents. Then, identify the physical quantity represented by the notation below the graph.   Homework Help ✎
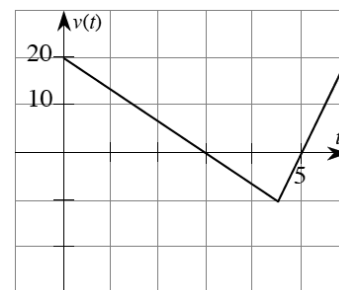

a. $s'(t)$ b. $\displaystyle\int_{a}^{b} r(t)\, dt$ c. $\displaystyle\int_{a}^{b} w(i)\, di$

4-100. The graph at right shows the velocity (in miles per hour) of a car during a road trip. At time $t = 0$, the car was on the Golden Gate Bridge heading north.
  Homework Help ✎

a. Find a function for $v(t)$.

b. How far north has the car traveled at 3 hours? At 4 hours?

c. Explain what happened to the car between $3 \le t \le 5$ hours.

d. Set up an integral to represent the displacement from $0 \le t \le 6$.

e. Set up an integral to represent the total distance from $0 \le t \le 6$.

4-101. During this course, we have studied the connection between distance and velocity.   Homework

a. What is the relationship between position, velocity, and the Fundamental Theorem?

b. Extension: Find a function to represent the car's position, $s(t)$, from problem 4-100.

4-102. Without graphing, analytically determine where the function $y = x^3 - 7x^2 + 15x$ - 2 is increasing. Check your answer with a graph.   Homework Help ✏

4-103. Differentiate the following functions. Determine if the function is differentiable for all reals.
 Homework Help ✏.

a. $y = \sin(x - 3)$

b. $f(x) = \begin{cases} 4 - x^2 & \text{for } x < 1 \\ (x-1)^3 + 3 & \text{for } x \geq 1 \end{cases}$

4-104. Write a Riemann sum to approximate $\int_0^2 (6^x)\,dx$ using n rectangles.   Homework Help ✏.

4-105. A function has a derivative of $f'(x) = 6x^2 + 12x - 7$.   Homework Help ✏.

a. If $f(0) = 0$, what was the original function?

b. If $f(-2) = 25$, what was the original function?

c. Describe how you found the constant of integration in parts (a) and (b).