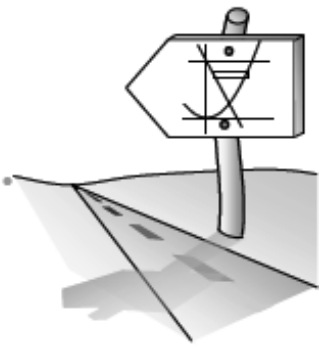
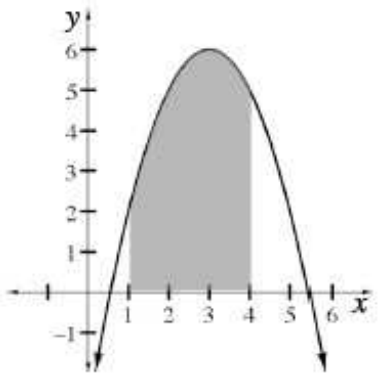
4.4.1 What is the area between curves?



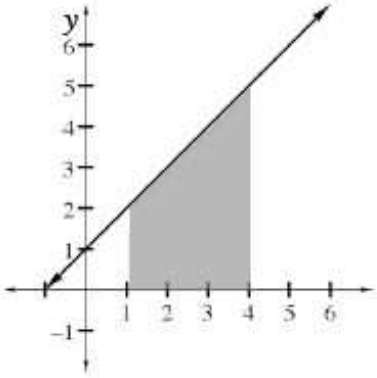
Area Between Curves

4-117. Set up integrals and find the exact areas for each of the shaded regions shown.

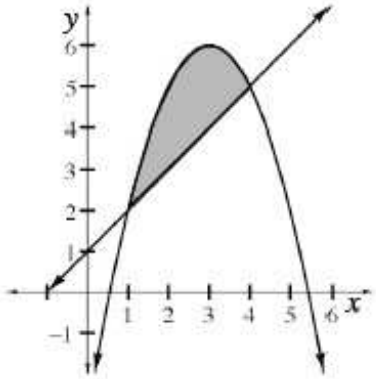
a. $f(x) = -x^2 + 6x - 3$



b. $g(x) = x + 1$

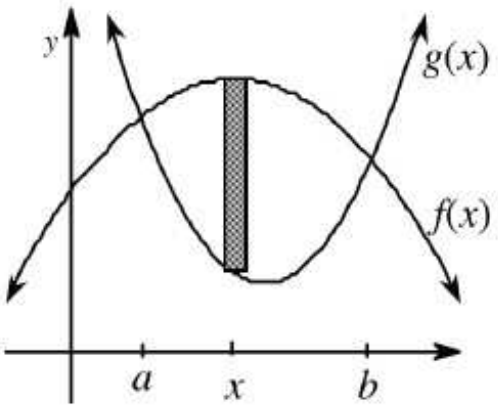


c. Between $f(x)$ and $g(x)$



4-118. AREA BETWEEN TWO CURVES

To find the area of the region between two curves, we take a limit of a sum of rectangle areas. A typical rectangle is shown in the diagram at right.



a. Copy the diagram on your paper. Label the rectangle with its length, width, and area.

b. Set up an integral that will add up rectangles between $a \le x \le b$.

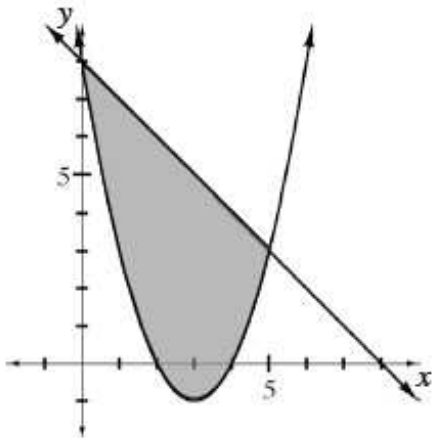
c. What do a and b represent?

4-119. Sketch the region enclosed by $f(x) = (x - 3)^2 - 1$ and $g(x) = -x + 8$, shown at right.

a. On your diagram, draw a typical rectangle. Label the rectangle with its dimensions and find its area.

b. Set up and evaluate an integral expression to find the area of the enclosed region. Check your solution with your graphing calculator.

c. Even though $f(x)$ dips below the x-axis, explain why we do not subtract off this portion.



4-120. Given: $f(x) = -x$ and $g(x) = x^2 - 6$

a. Set up and evaluate an integral to find the area bound by the curves in Quadrant IV.

b. Explain why the area is positive even though the graphs are below the x-axis.

4-121. Find the area of the enclosed regions below. A complete solution includes:

- A sketch with the shaded region.
- A typical rectangle with width and length labeled.
- An integral expression to add up the areas of all rectangles.
- An analytical solution, checked with a graphing calculator.

a. The area between $y = -(x - 3)^2 + 9$ and $y = x + 6$.

b. The area between $y = \sin x$ and $y = x^2 - 1$.

4-122. Describe how to find the bounds of integration when calculating the area between two curves.

4-123. Examine the following integrals. Consider the multiple tools available for evaluating integrals and use the best strategy for each. After evaluating the integral, write a short description of your method. Homework Help ✎

a. $\int_{-5}^{-2} \frac{3m^3+2m^2-9m}{m}\,dm$

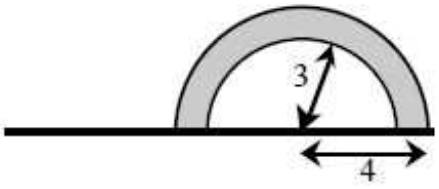
b. $\int_{-1}^{2} t(2t+3)dt$

c. $\int_{-4}^{-1} \left(1+\frac{1}{x}\right)^2 dx$

d. $\int_2^3 (ax+b)dx$

4-124. The area under the curve $f(x)$ can be found using the function $F(x) = 3(x-4)^3 + 6$. What is $f(x)$? Explain how this is an application of the Fundamental Theorem of Calculus. Homework Help ✎

4-125. A horizontal flag is shown below. The radius of the outer semicircle is 4, while that of the inner semicircle is 3. Homework Help ✎



a. Imagine rotating the flag about its pole and describe the resulting three-dimensional figure. Draw a picture of this figure on your paper.

b. Find the volume of the rotated flag.

4-126. Sketch $g(x)$ below and determine if it is differentiable at $x = 2$. Homework Help ✎

$$g(x) = \begin{cases} (x-1)^2 & \text{for } x < 2 \\ 2\sin(x-2)+1 & \text{for } x \geq 2 \end{cases}$$

4-127. As a log falls in a waterfall, its velocity is $v(t) = -32t - 18$ in feet per second. The position of the log at time $t = 0$ was at the top of the waterfall, 500 feet above sea level. Homework Help ✎

a. Where is the log after 1 second? 2 seconds? 3 seconds?

b. Where is the log after t seconds? This is the position function $s(t)$. What is its relationship to $v(t)$?

4-128. Write a complete set of approach statements for the following functions. Also, name any end behavior functions. Homework Help ✎

a. $f(x) = \frac{x^2-2x-3}{x-2}$

b. $f(x) = \frac{\cos x}{x}$

4-129. Does $\frac{d}{dx}((x-3)(2x+9)) = \frac{d}{dx}(x-3) \cdot \frac{d}{dx}(2x+9)$? Test your conjecture. Homework Help ✎

4-130. Evaluate. Homework Help ✎

a. $\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9}$

b. $\lim_{h \to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h}$

c. $\lim_{x \to \infty} \frac{2\sqrt{x}+1}{5-\sqrt{x}}$

d. $\lim_{x \to \infty} \cos x$