# 4.4.2 How do I find the area between 3 curves?

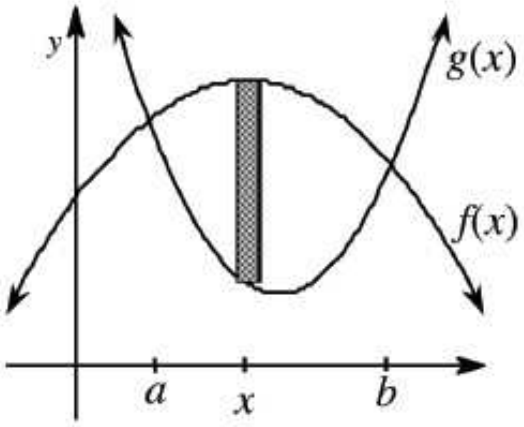
## More Area Between Curves

**4-131.** Find the area of the enclosed regions below. Review problem 4-117 for a description of a complete solution.

a. The area between $y = -2(x^2 - 1)$ and $y = -x^2 + 1$.

b. The area between $y = \sin x$ and $y = \frac{3}{4}x - 7$ for $\pi \le x \le 2\pi$.

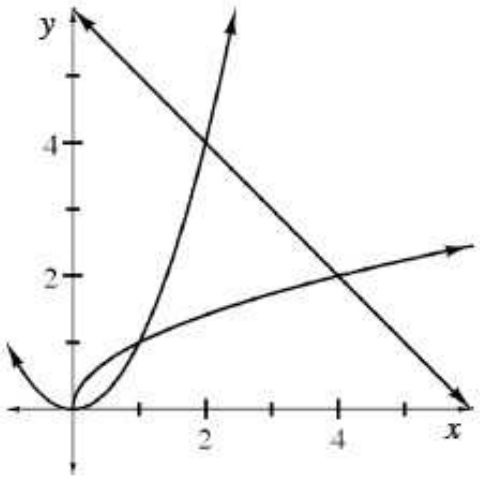
**4-132.** What would the result be if we calculated $\int_a^b (g(x) - f(x))dx$ instead of $\int_a^b (f(x) - g(x))dx$ for the functions $f(x)$ and $g(x)$ shown at right? Explain your thinking.




**4-133.** Examine the area of the region formed by the equations $y = x^2$, $y = \sqrt{x}$, and $y = -x + 6$ shown at right.

a. Explain why you cannot use one integral to find the area of the entire region.

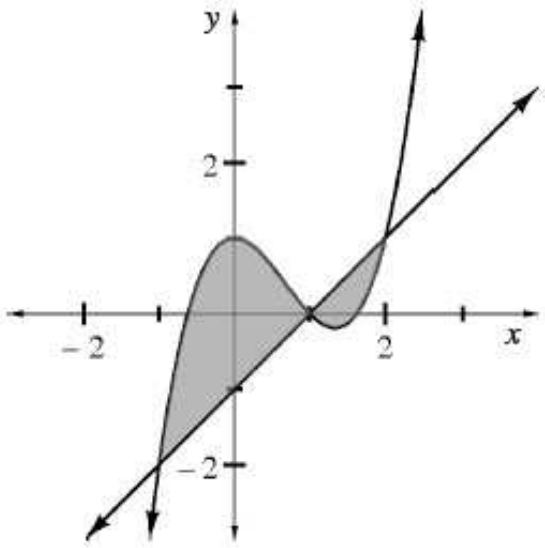
b. Write an expression using two integrals that would find the total area of the region using typical rectangles that are vertical. Then, evaluate the integrals and find the area.



**4-134.** Examine the region formed by the equations $f(x) = x - 1$ and $g(x) = x^3 - 2x^2 + 1$, shown at right.

a. Explain why this region requires two integrals.

b. Write and evaluate an integral expression to find the area between the curves. Check your answer with your graphing calculator.





**4-135.** Examine the integrals below. Consider the multiple tools available for evaluating integrals and use the best strategy for each. After evaluating the integral, write a short description of your method. Homework Help

a. $\int_0^5 \left( |x-2| + 3 \right) dx$

b. $\int \left( \frac{4}{m^3} - 3\cos m \right) dm$

c. $\int_1^2 x^x \, dx$

d. $\int \pi^2 \, dx$

e. $\int \sqrt{1-x^4} \, dx$

**4-136.** Draw a flag that would generate the same volume no matter if it were rotated about the $x$- or $y$-axis. Is there more than one possible shape of flag that meets this requirement? Does your flag have any special property that ensures these equal volumes of rotation? Homework Help

**4-137.** Without your calculator, evaluate the following limits. Homework Help

a. $\lim_{x\to 3^+} \sqrt{x-3}$

b. $\lim_{x\to\infty} \frac{x^2-2x+1}{x^3}$

c. $\lim_{x\to\pi} \frac{\cos x+1}{x-\pi}$

**4-138.** Use your calculator to approximate the following limits to two decimal places. Homework Help ✎

a. $\lim_{x\to 1} \frac{2^x-2}{3^x-3}$

b. $\lim_{x\to 0} \frac{1-\cos x}{x^2}$

**4-139.** For $f(x) = \sin(x^2)$, $g(x) = \sqrt{x-2}$, and $h(x) = \frac{1}{x}$, find the following functions and their domains. Homework Help ✎

a. $f(g(x))$

b. $f(g(h(x)))$

c. $h(f(g(x)))$

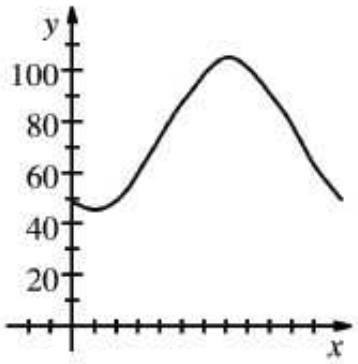
d. $h(h(x))$

**4-140.** Find the equation of the line that is: Homework Help ✎

a. Tangent to the function $y = \frac{3x^2-12}{x+2}$ at $x = 1$.

b. Normal to the tangent line at $x = 1$.

**4-141.** The annual cycle of daily high temperatures in Cabanaville is shown in the graph below from January 1st to December 31st. The $x$-axis is marked in segments of 30 days and the $y$-axis represents temperature in °F. Homework Help ✎



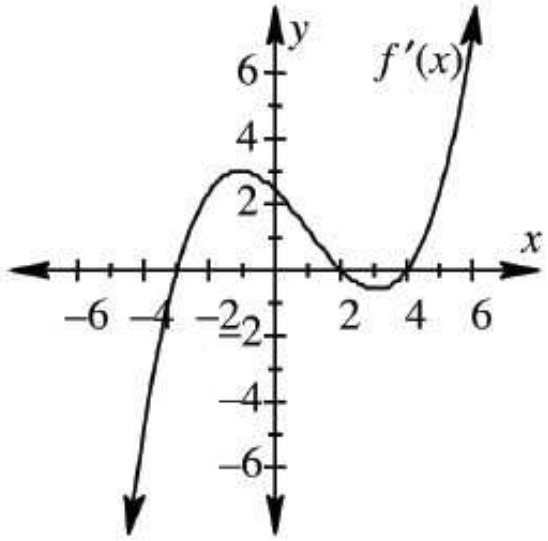

a. Approximately when is Cabanaville at its hottest? The coldest? How can you tell?

b. When is the temperature changing the fastest? What is this called?

**4-142.** Find $a$ such that $f(x)$ is differentiable at $x = 1$. Homework Help ✎

$$f(x)=\begin{cases}(x+2)^2-3 & \text{for } x<1\\ a\sin(x-1)+6 & \text{for } x\geq 1\end{cases}$$

**4-143.** The graph of $f'(x)$ is shown below. Find the intervals where $f(x)$ is increasing, decreasing, concave up, and concave down. Homework Help ✎




**4-144.** If $n$ is a positive integer, write a definite integral to represent. Homework Help ✎

$$\lim_{n\to\infty} \frac{2}{n}\left[\left(1+\frac{2}{n}\right)^2+\left(1+\frac{4}{n}\right)^2+\ldots+\left(1+\frac{2n}{n}\right)^2\right]$$