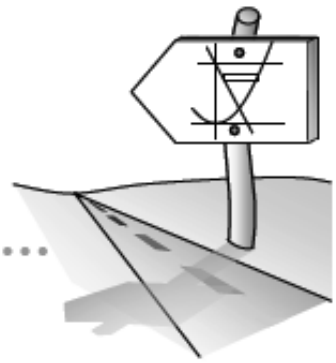
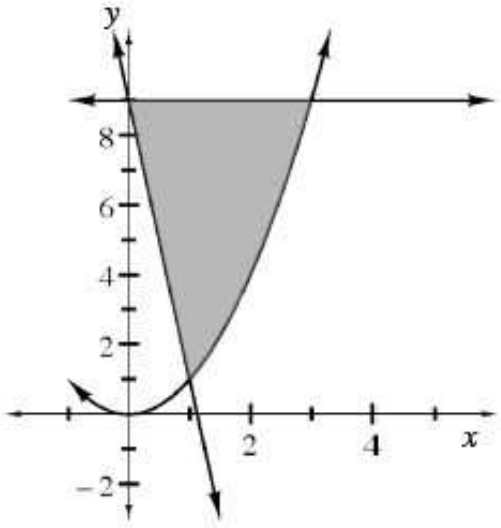
4.4.3 Should I sketch rectangles horizontally or vertically?



Multiple Methods for Finding Area Between Curves

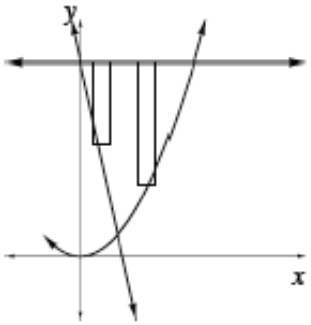
4-145. Adam, Becky, and Cathy were each working on the problem of finding the area bounded by the curves $y = x^2$, $y = 9$, and $y = -8x + 9$. Each approached the problem using different methods as shown below. Lesson 4.4.3 Resource Page



a. Label the dimensions of the typical rectangle in each diagram.

b. Describe the technique each student used. Decide if the method is valid. Then compute each integral to determine if each gives the same solution.

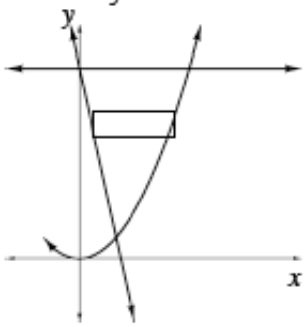
A. Adam's Method



Area =

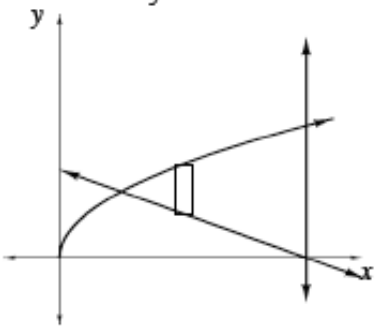
$$\int_0^1 (9-(-8x+9))\,dx + \int_1^3 (9-x^2)\,dx$$

B. Becky's Method



$$\text{Area} = \int_1^9 \left(\sqrt{y} - \frac{9-y}{8}\right) dy$$

C. Cathy's Method



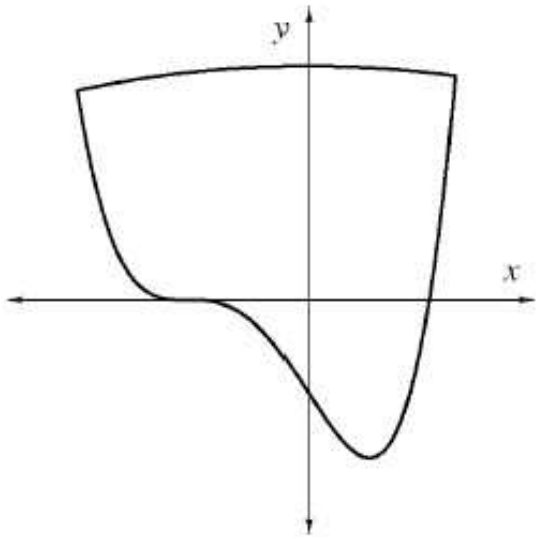
$$\text{Area} = \int_1^9 \left(\sqrt{x} - \frac{9-x}{8}\right) dx$$

c. **4-146.** Choose a method above to find the area enclosed in the first quadrant by $y = \sqrt{x}$ and $y = 4$. Be prepared to present your solution and describe your method to the class.

d. **4-147.** Find the area bound by the curves $y = 2\sqrt{x}$ and $y = x$ in the first quadrant in terms of y.

e. **4-148.** Find the total area enclosed by the functions $y = \sin x$ and $y = -\sin x$ for $0 \le x \le 2\pi$. Be prepared to present your solution and describe your method to the class.

f. **4-149.** FUNKY DESK
The Funky Furniture Company has designed a new desk for schools. The desktop is formed by the region contained by the functions:



$$f(x) = \frac{1}{512}x^4 + \frac{1}{32}x^3 - 2x - 8$$
$$g(x) = -\frac{2}{225}x^2 + 20$$

a. The elbow room is the distance from the x-axis to the lowest point on the curve. How much elbowroom is available on the desk if x is measured in inches?

b. Sketch the region on your paper. Draw and label a typical rectangle that could be used to find the area of the desk.

c. Set up and evaluate the integral to find the area of the desk.

4-150. Examine the following integrals. Consider the multiple tools available for evaluating integrals and use the best strategy for each. After evaluating the integral, write a short description of your method.
Homework Help ✎

a. $\int_{-1}^{1} \sqrt{x^2}\,dx$

b. $\int \left(8x^3 - \frac{1}{2}x\right)dx$

c. $\int_{1}^{5} \frac{3x^2 - 5x - 2}{3x + 1}\,dx$

d. $\int \left[\frac{d}{dx}(y)\right]dx$

4-151. Define possible functions $f(x)$ and $g(x)$ so that $h(x) = f(g(x))$. Remember $f(x) \neq x$, and $g(x) \neq x$.
Homework Help ✎

a. $h(x) = \sqrt[5]{\cos x}$

b. $h(x) = (3x\cos(x^2))^3$

c. $h(x) = 1$

d. $h(x) = x$

4-152. Find the value for a below so that $f(x)$ is continuous at $x = 3$. Homework Help

$$f(x) = \begin{cases} |4 - 3x| & \text{for } x < 3 \\ ax^2 + 2 & \text{for } x \geq 3 \end{cases}$$

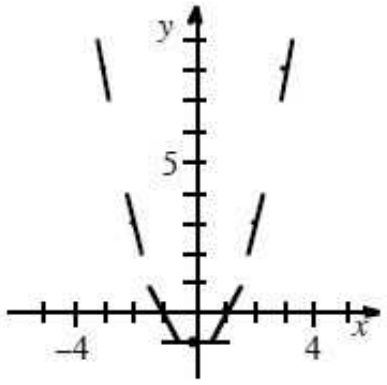
4-153. Find the linearization of $y = 4 - 2x^2$ at (1, 2). Then, use it to approximate y when $x = 1.15$. Homework Help

4-154. Set up an integral and compute the area of the enclosed regions below. You may use your graphing calculator. Homework Help

a. The area between $y = \sin x$ and $y = (x - 1)^4 - 1$.

b. The area between $y = x(x - 3)$ and $y = \sqrt{x}$.

4-155. Theresa loves tangents! This time, she has drawn lots of tangents and erased her original function. Find her original function. Homework Help



4-156. For the following functions, find an end behavior function. Explain your method. Homework Help

a. $y = \frac{6}{x} + 2x^2$

b. $y = \frac{\sin x}{x}$

c. $y = \frac{x^2 - 3x - 10}{x^2 + 1}$

4-157. For the functions listed in parts (a) through (c) of problem 4-156, find $\lim_{x \to \infty} y$. Describe the connection between the limit and the end behavior. Homework Help ✎