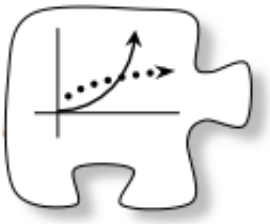
5.1.1 How can I "undo" a function?

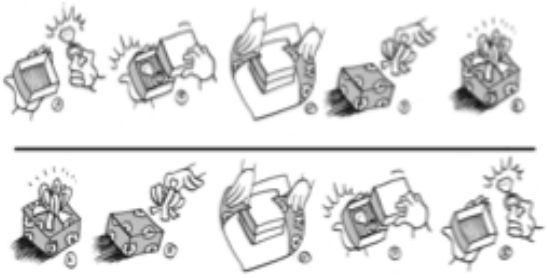


"Undo" Equations

Have you ever heard the expression, "She knows it forward and backward," to describe someone who understands an idea deeply? Often, being able to reverse a process is a way to show how thoroughly you understand it. Today you will reverse mathematical processes, including functions. As you worktoday, keep these questions in mind:

How can I "undo" it?

How can I justify each step?



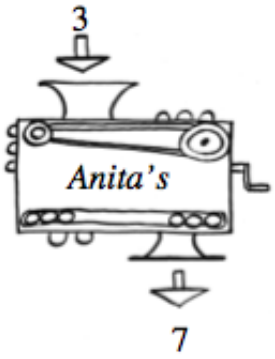
5-1. GUESS MY NUMBER

Today you will play the "Guess My Number" game. Your teacher will think of a number and tell you some information about that number. You will try to determine your teacher's number. (You can use your calculator or paper if it helps.) When you think you know the number, sit silently and do not tell anyone! Be sure to give others a chance to figure it out!

For example your teacher might say: "*When I add 4 to my number and then multiply the sum by 10, I get −70. What is my number*?"

Your task will be to find the number and explain your reasoning.

5-2. A picture of Anita's function machine is shown at right. When she put 3 into the machine, 7 came out. When she put in 4, 9 came out, and when she put in −3, −5 came out.



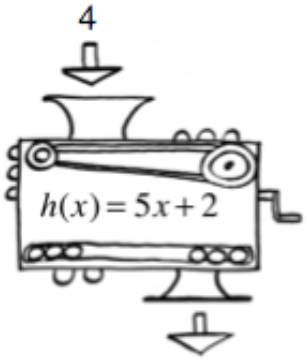
a. Make a table to organize the inputs and outputs from Anita's function machine. Explain in words what this machine is doing to the input to generate an output.

b. Anita's function machine suddenly starts working backwards: it is pulling outputs back up into the machine, reversing the machine's process, and returning the original input. If 7 is pulled back into this machine, what value do you think will come out of the top? Anita sets up her new backwards function machine and enters the other ouputs. What would you expect to come out the top if 9 is entered? If −5 is entered? Explain.

c. Record the inputs and outputs of the backwards function machine in a table. Record the numbers going in as x, and the numbers coming out as y. Explain in words what Anita's backwards function machine is doing.

d. Write equations for Anita's original function machine and for her backwards machine. How are the two equations related?

5-3. The function machine at right follows the equation $h(x) = 5x + 2$.



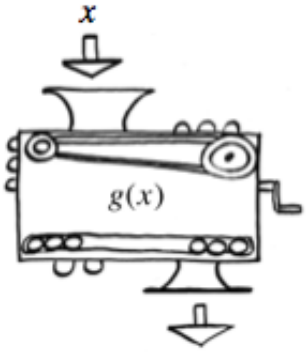
a. If the crank is turned backwards, what number should be pulled up into the machine in order to have a 4 come out of the top?

b. Keiko wants to build a new machine that will undo what $h(x)$ does to an input. What must Keiko's machine do to 17 to undo it and return a value of 3?

c. An "undo" function is called an inverse and has the notation $h^{-1}(x)$. Note that the -1 is not a negative exponent. It is the mathematical symbol that indicates the inverse function of $h(x)$. Write an equation for $h^{-1}(x)$, the "undo" function machine.

d. Choose a value for x. Then find a strategy to show that your equation, $h^{-1}(x)$, undoes the effects of the function machine $h(x)$.

5-4. Keiko was working with a new function, $g(x)$. He wrote down the following steps for $g(x)$:



- Add 5.
- Divide by 2.
- Cube it. (Find the third power.)
- Multiply by 6.

a. What is the equation for $g(x)$? What is the output when 3 is put in?

b. Help Keiko write down the steps (in words) of the inverse machine, $g^{-1}(x)$, and then write its equation.

c. Verify that your equation in part (b) correctly "undoes" the output of $g(x)$ in part (a).

5-5. Find the inverse equations for each of the functions below. Use function notation. Justify that each inverse equation works for its function.

a. $f(x) = 3x - 6$

b. $g(x) = x^3 - 5$

c. $p(x) = 2(x + 3)^3$

d. $t(x) = \frac{10(x-4)}{3}$

5-6. Each team member should choose one function and its inverse from the previous problem. Then they should

create a graph and a table for each pair. Be sure to graph the function and its inverse equation on the same set of axes.

When each person in your team has finished, put everyone's work into the middle of the workspace. Describe what relationships you see between the representations of a function and its inverse equation.

5-7. LEARNING LOG

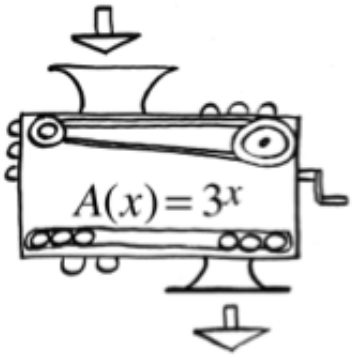
What strategies did your team use to find inverse equations? How can you be sure that the inverse equations you found are correct? Discuss this idea and then write a Learning Log entry about the strategies you have for finding inverse equations and checking that they work. Title this entry "Finding and Checking Inverse Equations" and label it with today's date.

5-8. Graph $y = \frac{1}{2}x - 3$ and its inverse function on the same set of axes. Help (Html5)⇔Help (Java)

a. What is the equation of the inverse function?

b. Does this graph, including both lines, have a line of symmetry? If so, what is the equation of the line of symmetry?

5-9. Antonio's function machine is shown at right. Help (Html5)⇔Help (Java)



a. What is $A(2)$?

b. If 81 came out, what was dropped in?

c. If 8 came out, what was dropped in? Be accurate to two decimal places.

5-10. Nossis has been working on his geometry homework and he is almost finished. His last task is to find a solution of $\sin(x) = 0.75$. Nossis cannot figure out what x could be! Explain how he can find a value for x and show that it works. Help (Html5)⇔Help (Java)

5-11. If $10^x = 10^y$, what is true about x and y? Justify your answer. Help (Html5)⇔Help (Java)

5-12. Solve each of the following equations for x. Help (Html5)⇔Help (Java)

a. $\frac{x}{3} = \frac{4}{5}$

b. $\frac{x}{x+1} = \frac{5}{7}$

c. $\frac{6}{15} = 2 - \frac{x}{5}$

d. $\frac{2}{3} + \frac{x}{5} = 6$

5-13. Sketch the solution of this system of inequalities. Help (Html5)⇔Help (Java)

$$y \geq x^2 - 5$$
$$y \leq -(x-1)^2 + 7$$

5-14. Jamilla collected data comparing the weight and cost of pieces of sterling silver jewelry. Her data is listed as (weight in ounces, cost in dollars): (5, 44.00), (8.5, 78.50), (12, 112.00), (10, 93.00), (7, 63.50), (9, 83.20). Help (Html5)⇔Help (Java)

a. Plot the data on a set of axes.

b. Use a ruler to draw a line that best approximates the data.

c. Determine the equation of the line of best fit drawn in (b).

d. Use your equation to predict the cost of a 50-ounce silver bracelet.

5-15. The angle of elevation of the sun (the angle the rays of sunlight make with the flat ground) at 10:00 a.m. is 29°. At that point, a tree's shadow is 32 feet long. How tall is the tree? Help (Html5)⇔Help (Java)