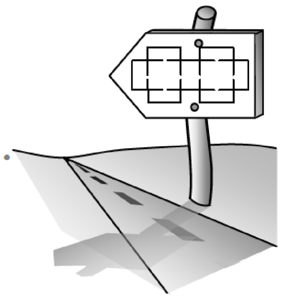
5.1.1 What is the starting position?

Distance, Velocity, and Acceleration Functions



5-1. SPITWADS, Part One
In her science class, Marisol often got into trouble for shooting spitwads. Because she was sneaky about it, she rarely got caught. One day, someone bumped her arm, causing a spitwad to shoot straight up in the air.

Out of curiosity, Marisol decided to find an equation for the height (above her head) of the spitwad as a function of time. She collected the data in the chart below.

Time (sec)	Height (feet)
0	0
1	48
1.5	60
3	48
3.5	28
4	0

a. Help Marisol by writing an equation for this data. Use h and t as your variables.

b. Marisol decided to find $\frac{dh}{dt}$ and $\frac{d^2h}{dt^2}$. What does this mean? Explain the notation.

c. Find $\frac{dh}{dt}$ and $\frac{d^2h}{dt^2}$. What does each tell you about the spitwad? What are the units?

d. Using $\frac{dh}{dt}$, explain why the acceleration is negative. Notice that acceleration is negative even when the spitwad is moving up (in the positive direction).

e. Notice that the acceleration is constant. What does this mean? What causes this constant acceleration?

5-2. Often coefficients of a position function give direct information about the movement of an object. Study these coefficients for a football kicked straight up in the air. Its elevation (in feet) at time t (in seconds) is represented by the function:

$$h(t) = -16t^2 + 30t + 2$$

a. Why is a quadratic an appropriate function for this situation?

b. What was the ball's starting position? Why is the starting position not zero?

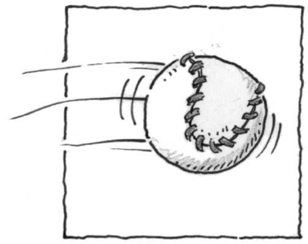
c. Find a function to represent the ball's velocity, $v(t)$, at time t. Then, use the velocity function to find the ball's initial velocity (when $t = 0$).

d. Is the initial velocity represented in the elevation equation $h(t) = -16t^2 + 30t + 2$? How?

e. Find a function to represent the ball's acceleration, $a(t)$, at time t. What are the units of acceleration? What is the significance of the result?

f. If the elevation function had been written using *meters* instead of feet, what should $a(t)$ be? What would the units for $a(t)$ be?

5-3. A baseball is thrown vertically with an initial velocity of 40 ft/sec and an initial height of 6 feet off the ground.



a. Assuming the only force on the baseball after it is thrown is gravity, what is $a(t)$?

b. Use $a(t)$ to find functions for $v(t)$ and $h(t)$. List the units for each.

c. How high was the ball at $t = 2$ seconds?

d. When was the ball at its maximum height? Describe your method.

5-4. On Earth, objects thrown into the air and falling objects have a position function of the form $s(t) = -16t^2 + bt + c$, where b and c are constants, and t is time.

a. Based on this form of a distance function, find the general functions for $v(t)$ and $a(t)$.

b. Study all three equations: $s(t)$, $v(t)$, and $a(t)$. Where does the initial velocity appear in $s(t)$? Where does the initial position appear in $s(t)$?

c. Study $a(t)$. What type of function is this? What does this tell you about the acceleration of a falling object on Earth?

Review & Preview

5-5. SPITWADS, Part Two
Marisol has more questions about her spitwad from problem 5-1. Homework Help ✎

a. How high is the spitwad after half a second? How fast is it traveling at that instant?

b. When is the spitwad 10 feet high? What is its velocity at those times?

c. As the spitwad is falling back toward Marisol, does it ever fall at 10 ft/sec? If so, when?

d. How long does the spitwad take to reach its highest point? How high does it go? Find your answer graphically and analytically.

5-6. Study the following information about $P(x)$: Homework Help ✎

$P(0) = 0$	$P'(2) = 0$	$P'(4) = -5$
$P'(0) = 1$	$P'(3) = -1$	$P'(6) = 0$

a. Use the information above to sketch a possible graph of $P(x)$.

b. Based on your sketch, give a reasonable estimate for $P'(1)$ and $P'(5)$.

5-7. You know that the first derivative, $f'(x)$, tells us the slope and the rate of change of $f(x)$. Homework Help ✎

a. What does the *second derivative*, $f''(x)$, tell you about $f'(x)$? What does $f''(x)$ tell you about $f(x)$?

b. Find $f'(x)$ and $f''(x)$ for $f(x) = x^3 + 3x^2 - 9x + 2$.

c. Use these derivatives to find $f''(1)$ and $f''(-2)$. Is $f(x)$ getting steeper or less steep at $x = 1$? At $x = -2$? Explain your reasoning.

d. The values in part (c) can be used to determine concavity. Where is $f(x)$ concave up? Where is $f(x)$ concave down?

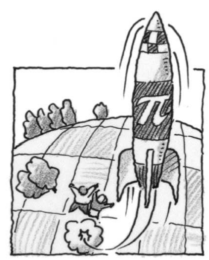
5-8. SNEAKY LIMITS
Lazy Lulu was examining the following limit. She recognized that this is the definition of the derivative as a limit. Homework Help ✎

$$\lim_{h\to 0} \frac{(5(x+h)^2-11(x+h)+7)-(5x^2-11x+7)}{h}$$

a. What is $f(x)$?

b. Use the Power Rule to find $f'(x)$.

c. Find the value of $\lim\limits_{h\to 0} \frac{(5(1+h)^2-11(1+h)+7)-(1)}{h}$. Explain and solve.



5-9. The Math Booster Club shot a rocket from the ground straight up into the air to celebrate π Day. The launcher shot the rocket with a starting velocity of 182 ft/sec. Find a function $s(t)$ for the height of the rocket. Then find its maximum height and the amount of time it was in the air. Homework Help ✎

5-10. Examine the following integrals. Consider the multiple tools available for evaluating integrals and use the best strategy for each. After evaluating the integral, write a short description of your method. Homework Help ✎

a. $\int_1^2 (15x^4 + \sqrt[3]{x})dx$

b. $\int_0^{\pi/3} 4\sin(y)dy$

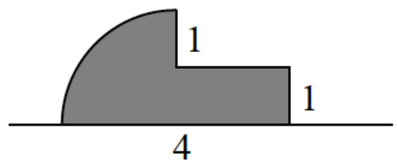
c. $\int_{-2}^3 (3x^2+1)dx$

d. $\int_{-2}^{2}\left(\sqrt{4-x^2}\right)dx$

e. $\int 2\cos(\theta-1)\,d\theta$

f. $\int \frac{2}{x^2}\,dx$

5-11. A horizontal flag, composed of a quarter-circle and straight line segments, is shown below.
Homework Help ✎



a. Imagine rotating the flag about its horizontal pole and describe the resulting three-dimensional figure. Draw a picture of this figure on your paper.

b. Find the volume of the rotated flag.