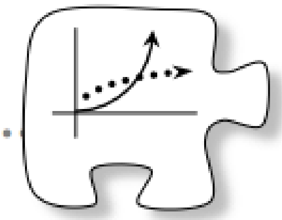
5.1.2 How can I find an inverse?



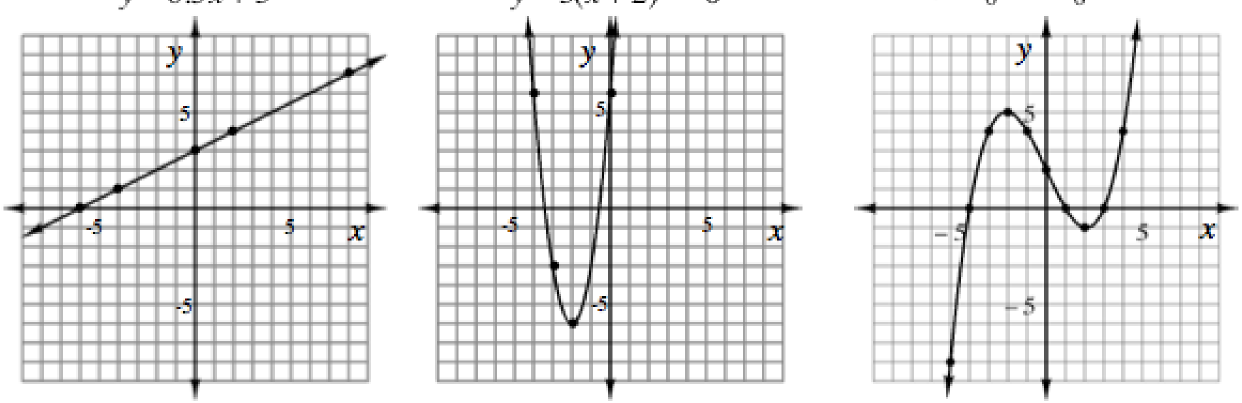
Using a Graph to Find an Inverse

What factors would you consider if you were thinking about buying a car? The first things that come to mind might be color or cost, but increasingly people are considering fuel efficiency (the number of miles a car can drive on a gallon of gas). You can think of the average number of miles per gallon that a car gets as a function that has *gallons* as the input and *miles traveled* as the output. A graph of this function would allow you to use what you know about the number of gallons in your tank to predict how far you could travel.

What would happen if you wanted to look at this situation differently? Imagine you regularly travel a route where there are many miles between gas stations. In this scenario, you would start with the information of the number of miles to the next filling station, and want to determine how many gallons of gas you would need to get there. In this case, you would start with the number of miles and work backwards to find gallons. Your new function would reverse the process.

5-16. In Lesson 5.1.1 you started with functions and worked backwards to find their inverse equations. Now you will focus on functions and their inverses represented as graphs. Use what you discovered yesterday as a basis for answering the questions below.

$y = 0.5x + 3$ $\qquad$ $y = 3(x+2)^2 - 6$ $\qquad$ $y = \frac{1}{6}x^3 - \frac{13}{6}x + 2$



a. Obtain a Lesson 5.1.2 Resource Page from your teacher and make a careful graph of each inverse equation on the same set of axes as its corresponding function. Look for a way to make the graph without finding the inverse equation first. Be prepared to share your strategy with the class.

b. Make statements about the relationship between the coordinates of a function and the coordinates of its inverse. Use $x \rightarrow y$ tables of the function and its inverse to show what you mean.

5-17. When you look at the graph of a function and its inverse, you can see a symmetrical relationship between the two graphs demonstrated by a line of symmetry.

a. Draw the line of symmetry for each pair of graphs in problem 5-16.

b. Find the equations of the line of symmetry for each graph.

c. Why do you think these lines make sense as the line of symmetry between the graphs of a function and its inverse relation?

5-18. The line of symmetry you identified in problem 5-17 can be used to help graph the inverse of a function without creating an $x \rightarrow y$ table.

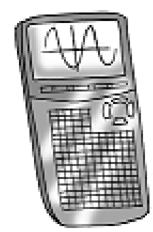
a. Graph $y = (\frac{x}{2})^2$ carefully on a full sheet of graph paper. Scale the x - and y-axes the same way on your graph.

b. On the same set of axes, graph the line of symmetry $y = x$.

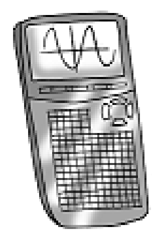
c. Trace over the curve $y = (\frac{x}{2})^2$ with a pencil or crayon until the curve is heavy and dark. Then fold your paper along the line $y = x$, with the graphs on the inside of the fold. Rub the graph to make a "carbon copy" of the parabola.

d. When you open the paper you should see the graph of the inverse. Fill in any pieces of the new graph that did not copy completely. **Justify** that the graphs you see are inverses of each other.

5-19. Your graphing calculator can also help you to graph the inverse of a function. Check your inverse graph from problem 5-18 by following your teacher's instructions to use the inverse-drawing feature of your graphing calculator. Was the inverse graph that you drew correct?



5-20. Find the equation of the inverse of $y = (\frac{x}{2})^2$. Is there another way you could write it? If so, show how the two equations are the same. **Justify** that your inverse equation undoes the original function and use a graphing calculator to check the graphs.



5-21. Consider your equation for the inverse of $y = (\frac{x}{2})^2$.

a. Is the inverse a function? How can you tell?

b. Use color to trace over the portion of your graph of $y = (\frac{x}{2})^2$ for which $x \geq 0$. Then use another color to trace the inverse of *only this part* of $y = (\frac{x}{2})^2$. Is the inverse of this part of $y = (\frac{x}{2})^2$ a function?

c. Find an equation for the inverse of the restricted graph of $y = (\frac{x}{2})^2$. How is this equation different from the one you found in problem 5-20?

5-22. Consider the function $f(x) = (x - 3)^2$.

a. How could you restrict the domain of $f(x)$ so that its inverse will be a function?

b. Graph $f(x)$ with its restricted domain and then graph its inverse on the same set of axes.

c. Find the equation of the inverse of $f(x)$ with its restricted domain.

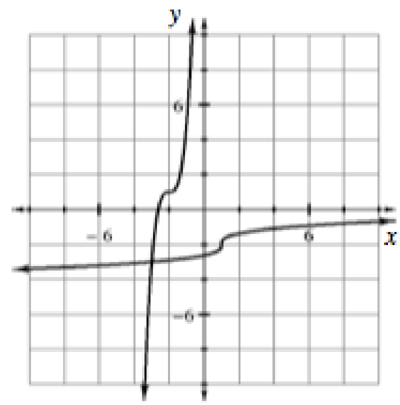
5-23. Is there a way to look at any graph to determine if its inverse will be a function? Explain. Find examples of other functions whose inverses are not functions.

5-24. Use graphs to find the inverses for the following functions. Label the graph of each function and its inverse with its equation.

a. $y = 5(x - 2)$

b. $y = 1 + \frac{2}{x}$

5-25. Look at the graph below of a function and its inverse. If $p(x)$ is a function and $q(x)$ is its inverse, can you tell which is which? Why or why not?



METHODS AND MEANINGS

MATH NOTES

Notation for Inverses

When given a function $f(x)$, the notation for the inverse of the function is $f^{-1}(x)$. Note that the –1 is not a negative exponent. It is the mathematical symbol that indicates the "undo" or **inverse** function of $f(x)$.

For example, if $f(x) = x^3 - 1$ then $f^{-1}(x) = \sqrt[3]{x+1}$.

This same inverse notation is used to identify the inverse of trigonometric functions. For example the inverse of $\sin(x)$ is written $\sin^{-1}(x)$.

Review & Preview

5-26. Make a graph of $f(x)=\frac{1}{2}(x-1)^3$ and then graph its inverse on the same set of axes. Help (Html5)⇔Help (Java)

5-27. Write the inverse equation for each of the following equations. Help (Html5)⇔Help (Java)

a. $y = 3x - 8$

b. $y=\frac{1}{2}x+6$

c. $y=\frac{x+6}{2}$

5-28. Solve the equation $3 = 8^x$ for x, accurate to the nearest hundredth (two decimal places). Help (Html5)⇔Help (Java)

5-29. Multiply each expression below. Help (Html5)⇔Help (Java)

a. $(x + 2)(x - 7)$

b. $(3m + 7)(2m - 1)$

c. $(x - 3)^2$

d. $(2y + 3)(2y - 3)$

5-30. Write the equation of a circle with a center at $(-3, 5)$ that is tangent to the y-axis (in other words, it touches the y-axis at only one point). Sketching a picture will help. Help (Html5)⇔Help (Java)

5-31. Perform the indicated operation to simplify each of the following expressions. In some cases, factoring may help you simplify. Help (Html5)⇔Help (Java)

a. $\frac{(x+2)(x-3)}{(x+1)(x-4)}\cdot\frac{(x+1)}{x(x+2)}$

b. $\frac{x^2+5x+6}{x^2-4}\cdot\frac{4}{x+3}$

c. $\frac{2x}{x+4}+\frac{8}{x+4}$

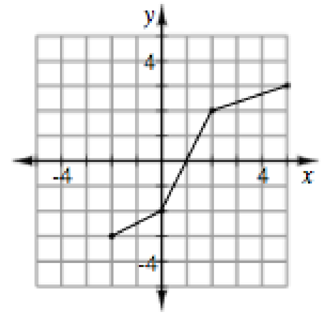
d. $\frac{x}{x+1}-\frac{1}{x+1}$

5-32. Barnaby's grandfather is always complaining that back when he was a teenager, he used to be able to buy his girlfriend dinner for only $1.50. Help (Html5)⇔Help (Java)

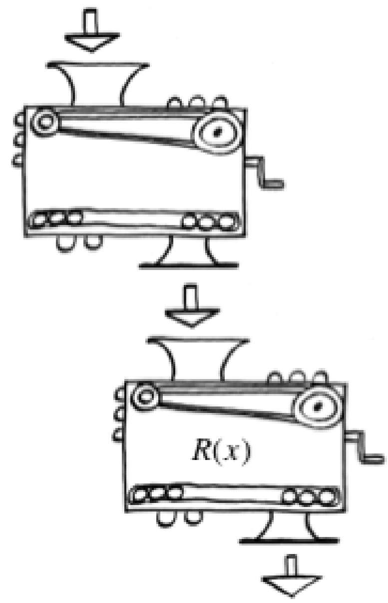
a. If that same dinner that Barnaby's grandfather purchased for $1.50 sixty years ago now costs $25.25, and the price has increased exponentially, write an equation that will give you the costs at different times.

b. How much would you expect the same dinner to cost in 60 years?

5-33. The function $f(x)$ is represented in the graph at right. Draw a graph of its inverse function. Be sure to state the domain and range for both $f(x)$ and $f^{-1}(x)$. Help (Html5)⇔Help (Java)



5-34. Lacey and Richens each have their own personal function machines. Lacey's machine, $L(x)$, squares the input and then subtracts one. Richens' function machine, $R(x)$, adds 2 to the input and then multiplies the result by three. Help (Html5)⇔Help (Java)



a. Write the equations that represent $L(x)$ and $R(x)$.

b. Lacey and Richens decide to connect their two machines, so that Lacey's output becomes Richens' input. If 3 is the initial input, what is the eventual output?

c. What if the order of the machines was changed? Would it change the output? **Justify** your answer.

5-35. Solve the system of equations below. Help (Html5)⇔Help (Java)

$$x - 2y = 7$$
$$6y - 3x = 33$$

a. What happened? What does this mean?

b. What does the solution tell you about the graphs?

5-36. Dana's mother gave her $175 on her sixteenth birthday. "*But you must put it in the bank and leave it there until your eighteenth birthday,*" she told Dana. Dana already had $237.54 in her account, which pays 3.25% annual interest, compounded quarterly. If she adds her birthday money to the account, how much money will she have on her eighteenth birthday if she makes no withdrawals before then? Justify your answer. Help (Html5)⇔Help (Java)

5-37. Multiply each expression below. Help (Html5)⇔Help (Java)

a. $(x + 4)(x - 14)$

b. $(2m + 5)(2m - 1)$

c. $(x-9)(x+9)$

d. $(3y+2)^2$

5-38. Calculate the x-intercepts for the graph of each function below. Help (Html5)⇔Help (Java)

a. $y=(x-2)(x+1)$

b. $y=2x^2+16x+30$

5-39. If $2^{x+4}=2^{3x-1}$, what is the value of x? Help (Html5)⇔Help (Java)