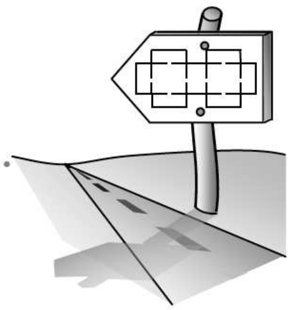
5.1.2 Which box has the largest volume?

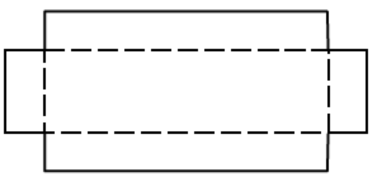
Optimization



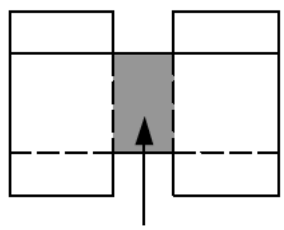
5-12. WRAP IT UP! Part One
Jaycee's Department store is preparing for the upcoming holiday season. Each year, customers line up to get their gifts wrapped and sometimes those gifts do not fit inside the boxes the company provides. This year, Jaycee's wants to redesign each of its boxes in order to maximize the volume. Lesson 5.1.2 Resource Pages

Jaycee's uses four basic box designs, shown below, with each outer rectangle measuring 7" by 4". Your task is to find the dimensions of each box design that will maximize the volume.

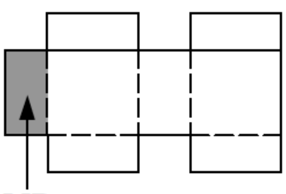
I. Open Box



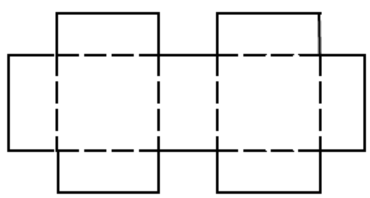
II. Strong Open Box



III. Closed Box



IV. Strong Closed Box



Each box uses multiple congruent square cutouts. For each box:

- Find a volume function $V(x)$ for any sized square cutout where x is the edge length of the square cutout.
- Find the physical limitations for the domain, x.
- Using a graph of your volume function, find the dimensions of the box that generates the maximum volume.

5-13. WRAP IT UP! Part Two
Explore a way to find the maximum volume without a graph:

a. What is the value of $V'(x)$ when the box has its maximum volume? Should that always be true?

b. Since the derivative of our cubic volume function is quadratic, there are potentially two values of x at which $V'(x) = 0$. Find both of these values without your calculator.

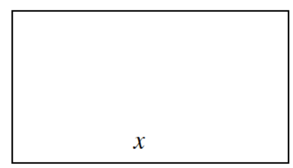
c. Describe a method that will determine the maximum and minimum values of a function.

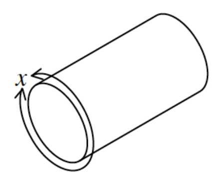
d. If setting the derivative equal to zero gives us multiple answers, how can we decide which maximizes and which minimizes the volume? Looking at the graph can help, but the graphing calculator may not show "funky functions" accurately. With your team, determine a method that will allow you to distinguish between a maximum and a minimum. How can you use calculus to *prove* whether a curve has reached its highest point or its lowest?

e. Sketch the graph of $f(x) = x^{2/3}$ on your calculator and observe that the minimum occurs when $x = 0$. What happens when you set $f'(x) = 0$? Adjust your description from part (c) to account for this situation.

Review & Preview

5-14. A rectangular piece of sheet metal with a perimeter of 50 cm is rolled into a cylinder with two open ends. Homework Help





a. Find the radius and height of the cylinder in terms of x.

b. Express the volume of the cylinder as a function of x.

c. Find the value of x that will maximize the volume and find the maximum value.

5-15. Find the second derivative with respect to x for each function. Homework Help

a. $y = (x - 10)^2$

b. $g(x) = -5x + 10$

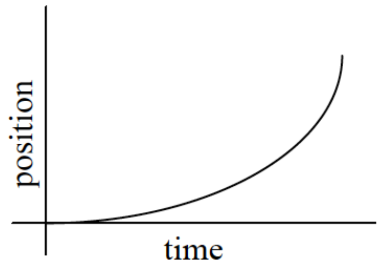
c. $y = 5x^8 - 3x^{-8}$

5-16. Without your graphing calculator, sketch $f(x)$ and algebraically determine where the function is increasing, decreasing, concave up, and concave down. List any maximums, minimums, and points of inflection. Homework Help

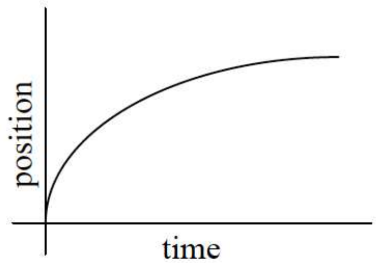
$$f(x) = \begin{cases} -(x-2)^2 + 3 & \text{for } x \le 2 \\ (x-3)^2 + 2 & \text{for } x > 2 \end{cases}$$

5-17. The curves below are pieces of position-time graphs. For each, sketch the graph of the second derivative and describe what it tells you about the motion. Homework Help ✎

a.



b.



5-18. Simplify each expression. Homework Help ✎

a. $\int \frac{(m^3-1)^2}{m^5}\,dm$

b. $\int_0^5 \left(\frac{d}{dx}\sqrt{x^2+11}\right)dx$

c. $\frac{d}{dx}\left[\int_0^5 \left(\frac{d}{dx}\sqrt{x^2+11}\right)dx\right]$

5-19. For $g(x)$ below, write and use your calculator to evaluate a Riemann sum to estimate $A(g, -4 \le x \le 4)$ for the functions below with 16 rectangles. Recall that a Riemann sum does not have to involve sigma notation. Homework Help ✎

a. $g(x) = x \sin^2 x$

b. $g(x) = \begin{cases} 3 & \text{for } x < 0 \\ x^2 + 3 & \text{for } x \ge 0 \end{cases}$

5-20. Investigate horizontal asymptotes for quotients of linear functions by examining the results of $\lim\limits_{x\to\infty}\left(\frac{ax+b}{cx+d}\right)$ for different values of a, b, c, and d. Homework Help ✎

a. In complete sentences, summarize your findings.

b. Find a function $f(x)$ that has a horizontal asymptote of $y = 3$ and a vertical asymptote $x = -5$