5.1.3 What can I do with inverses?

Finding Inverses and Justifying Algebraically

In this chapter you first learned how to find an inverse by undoing a function, and then you learned how to find an inverse graphically. You and your team may also have developed other strategies. In this lesson you will determine how to find an inverse by putting these ideas together and rewriting the equation. You will also learn a new way to combine functions that you can use to decide whether they have an inverse relationship.

5-40. Consider the table at right.

x	y
1	–5
3	7
5	19
7	31

a. Write an equation for the relationship represented in the table.

b. Make a table for the inverse.

c. How are these two tables related to each other?

d. Use the relationship between the tables to find a shortcut for changing the equation of the original function into its inverse.

e. Now solve this new equation for y.

f. **Justify** that the equations are inverses of each other.

5-41. Find the inverse function of the following functions using your new algebraic method, clearly showing all your steps.

a. $y = 2(x-1)^3$

b. $y = \sqrt{x-2} + 3$

c. $y = 3\left(\frac{x-9}{2}\right) + 20$

d. $y = \frac{4}{3}(x-1)^3 + 6$

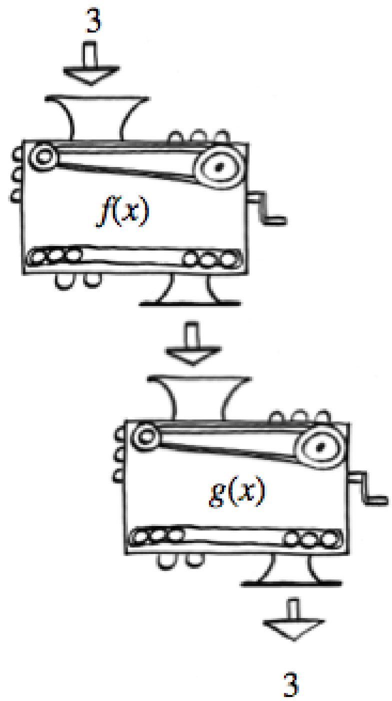
5-42. Adriena's strategy for checking that the functions $f(x)$ and $g(x)$ are inverses is to think of them as stacked function machines. She starts by choosing an input to drop into $f(x)$. Then she drops the output from $f(x)$ into $g(x)$. If she gets her original number, she is pretty sure that the two equations are inverses.

a. Is Adriena's strategy sufficient? Is there anything else she should test to be sure?

b. With your team, select a pair of inverse equations from problem 5-41, name them $f(x)$ and $g(x)$, then use

Adriena's ideas to test them.

c. Adriena wants to find a shortcut to show her work. She knows that if she chooses her input for $f(x)$ to be 3, she can write the output as $f(3)$. Next, $f(3)$ becomes the input for $g(x)$, and her output is 3. Since $f(3)$ is the new input for $g(x)$, she thinks that she can write this process as $g(f(3)) = 3$. Does her idea make sense? Why or why not?



d. Her friend Cemetra thinks she could also write $f(g(3))$. Is Cemetra correct? Why or why not.

e. Will this strategy for testing inverses work with any input? Choose a variable to use as an input to test with your team's functions, $f(x)$ and $g(x)$.

5-43. Statler, Adriena's teammate, is always looking for shortcuts. He thinks he has a way to adapt Adriena's strategy, but wants to check with his team before he tries it. "*If I use her strategy but instead of using a number, I skip a step and put the expression $f(x)$ directly into $g(x)$ to create $g(f(x))$, will I still be able to show that the equations are inverses*?"

a. What do you think about Statler's changes? What can you expect to get out?

b. Try Statler's idea on your team's equations, $f(x)$ and $g(x)$.

c. Describe your results.

d. Does Statler's strategy show that the two equations are inverses? How?

5-44. Trejo says that if you know the x-intercepts, y-intercepts, domain, and range of an equation then you automatically know the x-intercepts, y-intercepts, domain, and range for the inverse. Hilary disagrees. She says you know the intercepts but that is all you know for sure. Who is correct? **Justify** your answer.

5-45. Adriena was finding inverses of some equations. Use Statler's strategy from Problem 5-43 to check Adriena's work below and test if each pair of equations are inverses of each other. If they are not, explain what went wrong and show how to get the inverse correctly.

a. $f(x)=\frac{3}{5}x-15$

$g(x)=\frac{5}{3}x+25$

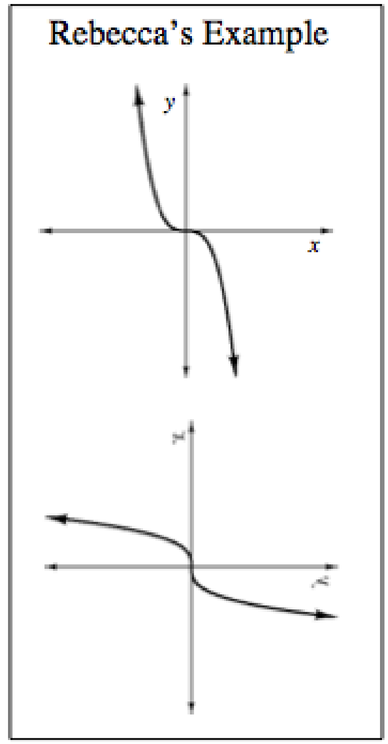
b. $f(x)=\frac{2(x+6)}{3}+10$

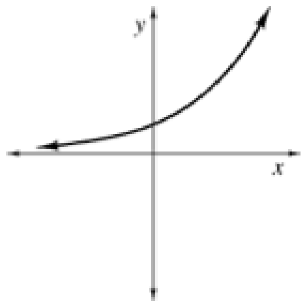
$g(x)=\frac{3}{2}x-21$

c. $e(x)=\frac{(x-10)^2}{4}$

$d(x)=4\sqrt{x}+10$

5-46. Rebecca thinks that she has found a quick way to graph an inverse of a function. She figures that if you can interchange x and y to find the inverse, she can interchange the x- and y-axes by flipping the paper over so that when she looks through the back the x-axis is vertical and the y-axis is horizontal as shown in the pair of graphs below left. Copy the graph below onto your paper and try her technique. Does it work? If so, do you like this method? Why or why not?





5-47. Make a personal poster that shows what you have learned about inverses so far. Choose an equation and its inverse then **justify** that your equations are inverses of each other using several representations.

METHODS AND MEANINGS

MATH NOTES

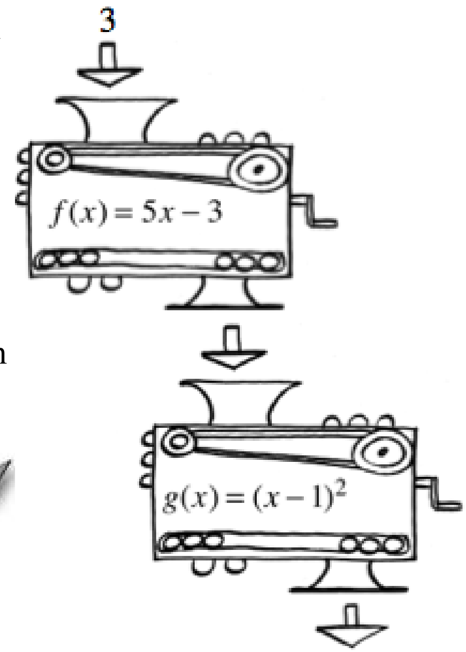
Composition of Functions

When we stack one function machine on top of another so that the output of the first machine becomes the input of the second, we create a new function, which is a **composition** of the two functions. If the first function is $g(x)$ and the second is $f(x)$, the composition of f and g can be written $f(g(x))$. (Note that the notations $f \circ g$ or $f \circ g(x)$ are used in some texts to denote the same composition.)

Note that the order of the composition matters. In general, the compositions $g(f(x))$ and $f(g(x))$ will be different functions.

Review & Preview

5-48. Two function machines, $f(x) = 5x - 3$ and $g(x) = (x - 1)^2$, are shown at right. Help (Html5)⇔Help (Java)



a. Suppose $f(3)$, (not $x = 3$), is dropped into the $g(x)$ machine. This is written as $g(f(3))$. What is this output?

b. Using the same function machines, what is $f(g(3))$? Be careful! The result is different from the last one because the order in which you use the machines has been switched! With $f(g(3))$, first you find $g(3)$, then you substitute that answer into the machine named $f(x)$.

5-49. This problem is a checkpoint for multiplying polynomials. It will be referred to as Checkpoint 5A.

Multiply and simplify each expression below.

a. $(x + 1)(2x^2 - 3)$

b. $(x + 1)(x^2 - 2x + 3)$

c. $2(x + 3)^2$

d. $(x + 1)(2x - 3)^2$

Check your answers by referring to the Checkpoint 5A materials.

If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 5A materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do problems like these quickly and easily.

5-50. Solve each of the following equations. Help (Html5)⇔Help (Java)

a. $\frac{3x}{5} = \frac{x-2}{4}$

b. $\frac{4x-1}{x} = 3x$

c. $\frac{2x}{5} - \frac{1}{3} = \frac{137}{3}$

d. $\frac{4x-1}{x+1} = x - 1$

5-51. Find the inverse of each of the following functions by first switching x and $y$and then solving for y. Help (Html5)⇔Help (Java)

a. $y = x^2 + 3$

b. $y = \left(\frac{1}{4}x + 6\right)^3$

c. $y = \sqrt{5x - 6}$

5-52. Complete the square (for x) to write the equation that follows in graphing form and sketch the graph of $x^2 + y^2 - 4x - 16 = 0$. What is the parent graph and how has it been transformed? Help (Html5)⇔Help (Java)

5-53. Ever eat a maggot? Guess again! The FDA publishes a list, the Food Defect Action Levels list, which indicates limits for "natural or unavoidable" substances in processed food (*Time*, October 1990). So in 100 grams of mushrooms, for instance, the government allows 20 maggots! The average batch of rich and chunky spaghetti sauce has 350 grams of mushrooms. How many maggots does the government allow in a batch? Help (Html5)⇔Help (Java)

5-54. Perform each operation below and simplify your results. Help (Html5)⇔Help (Java)

a. $\frac{x^2 + 4x + 3}{x^2 + 3x} \cdot \frac{3x}{x + 1}$

b. $\frac{y^2}{y + 4} - \frac{16}{y + 4}$

c. $\frac{x^2 + x}{x^2 - 4x - 5} \div \frac{3x^2}{x - 5}$

d. $\frac{x^2 - 6x}{x^2 - 4x + 4} + \frac{4x}{x^2 - 4x + 4}$