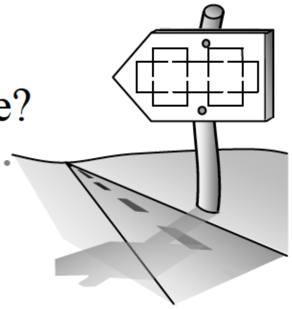
5.1.3 What information do f' and f'' give?



Using the 1st and 2nd Derivatives

5-21. RADIO DAZE

Avantika is very excited. Her favorite radio station, KNRD, is having a contest and she thinks she can win. The grand prize includes unlimited rides on the awesome Rotating Flags roller coaster!

Avantika listens intently as her favorite announcer, Ima Geek, reads the question:

> "Our mystery function today has these properties," she starts.
> "$f(2) = 5$,
> $f'(2) = 0$, $f'(x) < 0$ for all x less than 2, and
> $f'(x) > 0$ for all x greater than 2.

Remember that all of our mystery functions are continuous and defined for all x. To win, you must describe a small portion of the function, including the coordinates of a key point. You must also describe something interesting about the graph at this point. We'll take the twelfth caller, but if your answer is incorrect, you will be barred from all future KNRD contests, so be careful."

Avantika badly needs your team's help with this. A picture is worth a thousand words, so a labeled sketch will be very helpful. Since Avantika will have to tell Ima the answer verbally, your team needs to describe the sketch in words as well.

5-22. Since Avantika answered the first question correctly with your help, she advanced to the second round.

"Our mystery function today has these properties," Ima says.
"$f''(3) < 0$, $f'(3) = 0$, and $f(3) = 2$.

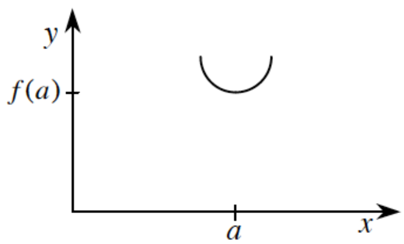
We'll take the n^{th} caller, where $n = 15$," she said in her nerdiest voice.

Avantika is really stumped. Can your team come to her rescue yet again?

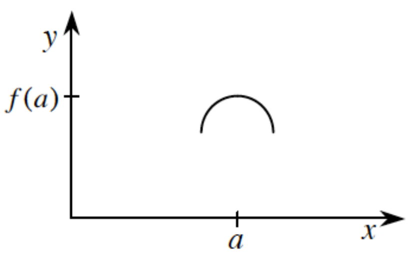
5-23. Ima Geek of KNRD is taking a week vacation and needs to have the rest of the graphs and clues ready for her replacement and father Kit Protector (Ima calls him Pa). She has all of the different graphs that will be used for the week, but she needs to generate a list of descriptions for each.

The hints need to describe the characteristics of $f'(x)$ at $x = a$ and for values of x to the left and right of a. The second round hints include characteristics of $f''(x)$ at $x = a$. The clues for $f(x)$ will be supplied by the announcer. For each graph, supply as many clues as possible for the contest.

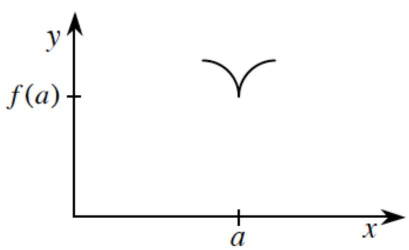
a.



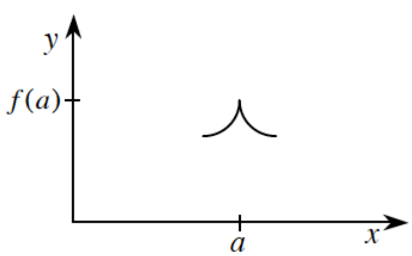
b.



c.



d.



5-24. MORE FUNKY FUNCTIONS
The first and second derivatives are not only tools to prove a function has a minimum or a maximum. They also are the most reliable tools to find minimums and maximums. To see why, consider $f(x) = 10x^{12} - 3x^{10}$.

a. Use your graphing calculator to graph the function. Make a sketch, including some numbers on the axes, and write your preliminary conclusions. Does the function have any local maxima or minima?

b. Without a graphing calculator, find the x-values where $f'(x) = 0$. xy-coordinate points where $f'(x) = 0$ or does not exist are called **critical points** because these points are 'critical' to the analysis.

c. Notice that in part (b) you found the x-values of the critical points. What about the y-values? Without a calculator, the y-coordinates of the critical points are difficult to find. However, the first derivative can help us imagine what the graph looks like. By observing how the sign of the first derivative changes at the critical points, you can tell whether they are local maxima or minima. Determine where the first derivative is positive and where it is negative. This is known as the **First Derivative Test for Extrema**.

d. Make a second sketch of the graph showing what it really looks like near the origin.

e. The second derivative also helps to determine the nature of critical points. At $x = 0.5$, for example, $f'(0.5) = 0$ and $f''(0.5) > 0$. What happens to the graph at $x = 0.5$?

f. As shown in part (e), the sign of the second derivative at a critical point can be a useful tool to distinguish between a maximum and a minimum. This is known as the **Second Derivative Test for Extrema**. Recall that a function is concave up when the second derivative is positive. Explain why it makes sense that the critical point on that interval must be a minimum.

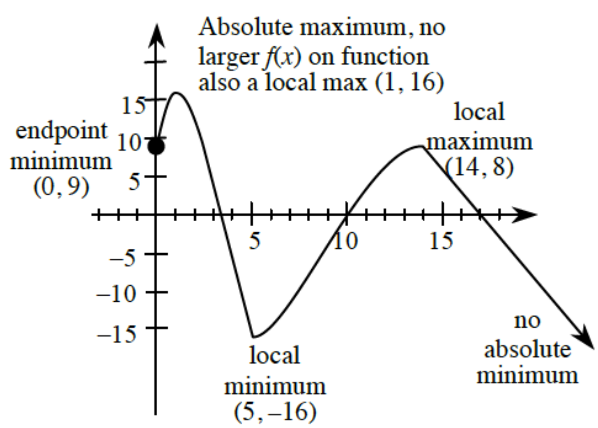
g. What happens to the graph at $x = -0.5$? Justify your answer in two different ways: use the first and second Derivative Tests.

h. What happens to the graph at $x = 0$? Is there a local maximum or local minimum? Use bothtests to check your answer. Do both tests answer the question? Which of the tests does not help answer the question?

i. Confirm your answer to part (d) using your calculator. Choose your graphing windows carefully.

MATH NOTES

Extreme Values



Often, we are interested in where a function takes a **maximum** or a **minimum** value. These values are also called **extreme values** or**extrema**.

There are three types of extrema: **local** (also called **relative**), **global** (also called **absolute**), and **endpoint**.

Endpoint extrema occur in problems where the domain is limited, either by instruction or by the conditions of an application problem. Examine the graph at right for examples of local and absolute extrema.

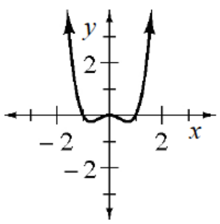
(The AP test currently uses the words "relative" and "absolute," but the words "local" and "global" appear regularly in college textbooks. It would be wise to be prepared for both possibilities.)

Extreme Value Theorem: If f is continuous on a closed interval $[a, b]$ then f has both an absolute minimum and an absolute maximum on the interval.

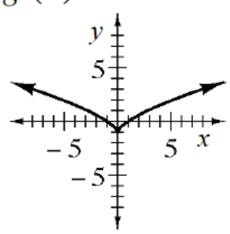
Review & Preview

5-25. Use the graphs of $f'(x)$ and $g'(x)$ below to determine the x-values of all local minimums, maximums, and points of inflection. Homework Help ✎

a. $f'(x)$



b. $g'(x)$



5-26. Examine the mystery functions described below. Homework Help ✎

a. $f(x)$ has all of the following properties: $f''(2) < 0$, $f'(2) = 0$ and $f(2) = 1$. Assuming the function is continuous, sketch a small portion of the mystery function near $x = 2$. Describe the function at this point.

b. A different mystery function has all of the following properties: $f'(5) = 0$, $f'(x) > 0$ for $4.98 < x < 5$, and $f'(x) < 0$ for $5 < x < 5.02$. Draw a sketch of $f(x)$ near $x = 5$ and state a conclusion.

5-27. Identify the maxima and minima of $f(x) = -\frac{1}{4}x^4 + 3x^3 - 10x^2 + 40$. Homework Help ✎

5-28. Find the area between $y = \sin(\pi x)$ and $y = x^2 - 3x$. Homework Help ✎

5-29. Kwok is standing on top of a 160-foot building and throws a tennis ball straight up with an initial velocity of 24 ft/sec. Homework Help ✎

a. How far up does the tennis ball go before it turns around and starts to come down?

b. How fast is it going when it hits the ground below?

5-30. Mr. Lyon was trying to find the points of inflection for $f(x) = 3x^{2/3}$. Since no value of x existed such that $f''(x) = 0$, he assumed there was no change in concavity. Andrew said that a point of inflection exists where the second derivative changes sign. Who is correct and why? Use the second derivative and the graph of $f(x)$ to verify your answer. Homework Help ✎

5-31. Graph $y = |x+1| + |x-2|$. For what value of x does y achieve its minimum value? What characteristic of this function makes the answer to this question unusual? Homework Help ✎

5-32. Examine the integrals below. Consider the multiple tools available for evaluating integrals and use the best strategy for each. After evaluating the integral, write a short description of your method. Homework Help ✎

a. $\int_1^4 (2x+1)^2\,dx$

b. $\int_{-\pi}^{\pi} x\cos x\,dx$

c. $\int (2\sin^2 x + 2\cos^2 x)\,dx$

d. $\int \left(\frac{t^2}{2} - \frac{2}{t^2}\right)dt$