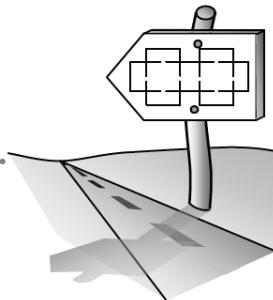


5.1.4 Did you check your endpoints?

..... Applying the 1st and 2nd Derivative Tests



5-33. Karen was determined to find a maximum for a function $f(x)$. She found the derivative of $f(x)$ and set it equal to 0. Kirt commented that although her method could find a maximum, it could also find a minimum!

- a. Explain (with a diagram) what Kirt means.
- b. Koy stated that she thinks there could be a maximum that Karen's method will not detect. How is that possible?
- c. Ms. Maximillion said that even if $f'(x) = 0$, it does not necessarily mean that there is a minimum or a maximum at x . Is that true? Draw a picture to justify your answer.
- d. Examine the following Math Notes box regarding critical points and the first and second derivative tests.

MATH NOTES

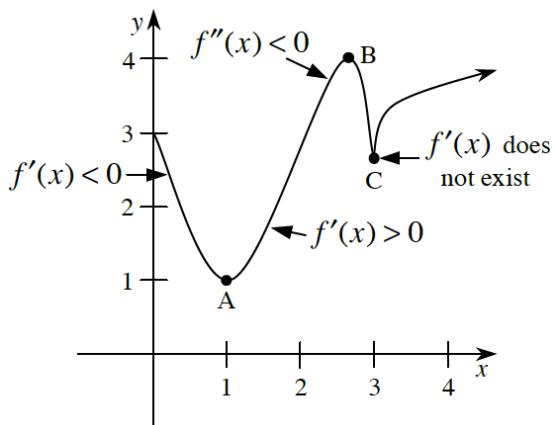


Critical Points and the Derivative Tests for Extrema

Although the condition $f'(x) = 0$ is often useful for finding relative extrema (points A and B in the diagram), we have seen that local extrema can also occur at points where $f'(x)$ does not exist (point C). Interior points (not endpoints) where maxima and minima might occur are called critical points. To test for global extrema, we must also consider the endpoints.

Critical points are points where the first derivative is either zero or does not exist.

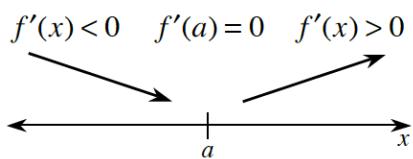
We can test whether a critical point is a local minimum or maximum using either the first or the second derivative.



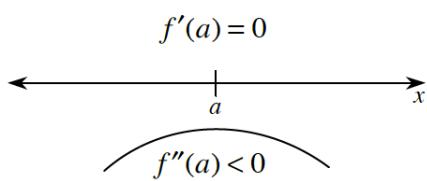
First Derivative Test: If the first derivative goes from negative (on the left) to positive (on the right) as we pass through a critical point, then the point is a local minimum. Points A and C show this situation in the graphic above. (There is a similar test for a local maximum.)

Second Derivative Test: If the first derivative is zero and the second derivative is negative at a critical point, then the point is a local maximum. Point B shows this situation. (There is a similar test for a local minimum.)

First Derivative Test



Second Derivative Test



5-34. Remembering all the information in the Math Notes box is not as hard as it may seem. Drawing pictures is very useful. You need to remember that a positive *first* derivative means an increasing *function* (positive slope) and a positive *second* derivative means increasing *slope* (concave up).

- What does a negative first derivative mean?
- What does a negative second derivative mean?
- State the first derivative test for a relative maximum.
- State the second derivative test for a relative minimum.

5-35. The second derivative can tell you more than just concavity and points of inflection. It can also allow you to determine when a tangent line will provide an under or over approximation.

- Sketch situations where the tangent will be an under and over approximation.

- b. What is significant about the second derivative in each situation?

5-36. ENERGY CRISIS

In January 2001, energy prices in California began to rise dramatically. The average cost per megawatt hour could be modeled by the function

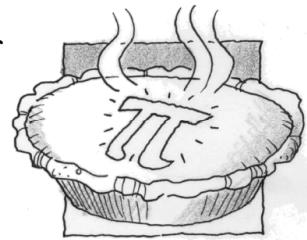
$$C(d) = 0.1d^3 - 4.2d^2 + 54d + 10$$

where d is the day in January and $C(d)$ is the average cost per megawatt.

- On which day in January was the average megawatt cost the highest? Be sure to justify your answer by using the appropriate test.
- Using the results from part (a), set an appropriate window that contains the function during the month of January. Graph the function and verify your solution to part (a). Alter your solution to part (a) if necessary. Explain what happened.

5-37. PI DAY PIES

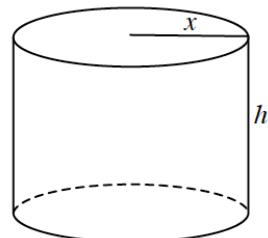
Every year, to celebrate Pi Day, the Math Club sells pies to students (Pican pies, of course). From past experience, they know that if they sell the pies for \$5.75 each, 314 pies will be sold. However, for every 25¢ they raise the price, they will sell approximately 10 fewer pies.



- How much should the Math Club charge per pie so that they make the most money? Solve this problem any way you want, but be sure to include an equation and graph in your solution
- If you have not done so already, determine how to use derivatives to help you answer this question. Explain completely.

5-38. The Tasty Tuna Company needs to design a can that will have a volume of 20 cubic inches and will minimize the amount of materials necessary to build the can. If the radius of the base of the can is x inches and the tangent is h inches:

- Find an expression for the total surface area of the can using both x and h .
- Find an expression for the volume of the can using x and h .
- Use your results from part (b) to find an expression for the surface area in terms of x .
- Find the dimensions that will minimize the surface area.



5-39. YELLOW SUBMARINE, Part One

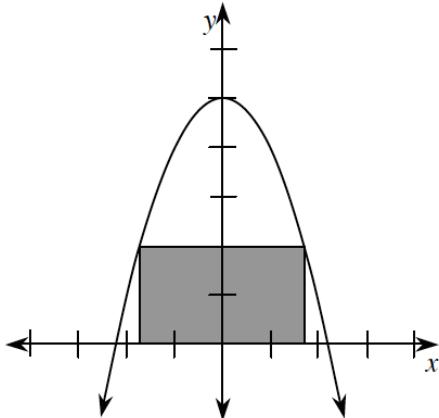
Ring O. Star was sick and tired of chasing blue meanies around the inside of his yellow submarine, so he decided to build a special detention chamber to hold the ones he caught. The first design he thought of

was a rectangular chamber with sides made of razor-sharp barbed wire. To save money, he decided to build the chamber up against a submarine wall - that way only three sides needed to be made out of barbed wire. He estimated the detention chamber should have 40 square meters of area. What dimensions should the chamber have so that Mr. Star can buy the least amount of barbed wire?



Homework Help ↗

- 5-40. A rectangle is bounded by the function $y = -x^2 + 5$ and the x -axis as shown below. Homework Help ↗



- If the base of the rectangle is $2x$, what is the height?
- What is the maximum area that the rectangle can obtain?
- If you made the rectangle from part (b) into a flag and spun it around the x -axis, what would be the resulting volume?
- Find x so that the volume of the rotated rectangle is a maximum.

- 5-41. Sketch a continuous, differentiable graph with only one absolute maximum and two relative minima. Homework Help ↗

5-42. The second derivative test is easy to apply once you have found the second derivative, but unfortunately it does not always work. The first derivative test works more often, but is harder to apply. Let's compare the two tests by investigating the graphs of $y = x^2$, $y = x^3$, and $y = x^4$ at the origin.

Homework Help ↗

- Use the first derivative test to investigate the behavior of $y = x^2$, $y = x^3$, and $y = x^4$ at the origin.
- Try to confirm your part (a) results using the second derivative test. When does the test work? What can you conclude if you know that the second derivative equals zero at the origin?

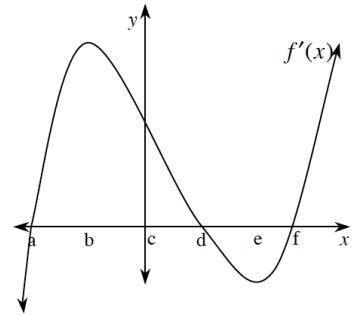
- 5-43. Using the graph of the slope function $f'(x)$ below, determine where the following situations occur for $f(x)$: Homework Help ↗

- Relative minima and maxima.
- Intervals at which $f(x)$ is increasing.

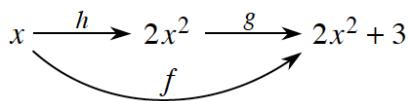
- c. Inflection points.
d. Intervals at which $f(x)$ is concave up and concave down.

5-44. A "CHAIN" OF FUNCTIONS

Remember that composite functions are made by "chaining" simpler functions together. For example, $f(x) = 2x^2 + 3$ is made by chaining the functions $h(x) = 2x^2$ and $g(x) = x + 3$.



It can be useful to show this idea with an arrow diagram:



Above is an arrow diagram for h followed by g , which would be written $f(x) = g(h(x))$. (Notice that the "inner" function is applied first.)

Let $h(x) = \tan x + 5$ and $g(x) = (x + 3)^2$. Draw an arrow diagram and determine the composite function for each $f(x)$ below. Simplifying is not required. [Homework Help](#) ↗

- a. $f(x) = g(h(x))$
b. $f(x) = h(g(x))$
c. $f(x) = g(g(h(x)))$
d. $f(x) = h(g(g(x)))$

5-45. Evaluate integrals below. Assume that a and b are constants. [Homework Help](#) ↗

- a. $\int (a^2) dx$
b. $\int (bu^2 - a) du$
c. $\int (am - b) dm$

5-46. Let $f(x) = x^2 + 3x$ and $g(x) = \sin x$. Match the expression in the left-hand column with its simplification in the right-hand column. [Homework Help](#) ↗

- | | |
|---------------|--------------------------|
| a. $f'(g(x))$ | A. $\sin^2 x + 3 \sin x$ |
| b. $f(g(x))$ | B. $\cos(x^2 + 3x)$ |
| c. $g(f(x))$ | C. $2 \sin x + 3$ |
| d. $g'(f(x))$ | D. $\sin(x^2 + 3x)$ |