5.2.2 What is a logarithm?

Defining the Inverse of an Exponential Function

You have learned how to "undo" many different functions. However, the exponential function has posed some difficulty. In this lesson, you will learn more about the inverse exponential function. In particular, you will learn how to write an inverse exponential function in $y =$ form.

5-68. SILENT BOARD GAME

Your teacher will put an $x \rightarrow y$ table on the board or overhead that the whole class will work together to complete. The table will be like the one below. See which values you can fill in.

x	8	32	$\frac{1}{2}$	1	16	4	3	64	2	0	0.25	−1	$\sqrt{2}$	0.2	$\frac{1}{8}$
$g(x)$	3		−1												

a. Describe an equation that relates x and $g(x)$.

b. Look back to the Ancient Puzzle in problem 5-57. If you have not already done so, use the idea of the Ancient Puzzle to write an equation for $g(x)$.

c. Why was it difficult to think of an output for the input of 0 or −1?

d. Find an output for $x = 25$ to the nearest hundredth.

5-69. ANOTHER LOGARITHM TABLE

Lynn was supposed to fill in this table for $g(x) = \log_5 x$. She thought she could use the log button on her calculator, but when she tried to enter 5, 25, and 125, she did not get the ouputs the table below displays. She was fuming over how long it was going to take to guess and check each one when her sister suggested that she did not have to do that for all of them. She could fill in a few more and then use what she knew about exponents to figure out some of the others.

x	$\frac{1}{25}$	$\frac{1}{5}$	$\frac{1}{2}$	1	2	3	4	5	6	7	8	10	25	100	125	625
$g(x)$		−1		0				1					2		3	

a. Discuss with your team which outputs can be filled in without a calculator. Fill those in and explain how you found these entries.

b. With your team, use your calculator to estimate the remaining values of $g(x)$ to the nearest hundredth. Once you have entered several, use your knowledge of exponent rules to see if you can find any shortcuts.

c. What do you notice about the results for $g(x)$ as x increases?

d. Use your table to draw the graph of $y = \log_5 x$. How does your graph compare to the graph of $y = 5^x$?

5-70. Find each of the values below, and then **justify** your answers by writing the equivalent exponential form.

a. $\log_2(32) = ?$

b. $\log_2(\frac{1}{2}) = ?$

c. $\log_2(4) = ?$

d. $\log_2(0) = ?$

e. $\log_2(?) = 3$

f. $\log_2(?) = \frac{1}{2}$

g. $\log_2(\frac{1}{16}) = ?$

h. $\log_2(?) = 0$

5-71. While the idea behind the Ancient Puzzle is more than 2100 years old, the symbol **log** is more recent. It was created by John Napier, a Scottish mathematician in the 1600's. "Log" is short for **logarithm**, and represents the function that is the**inverse of an exponential function**. You can use this idea to find the inverse equations of each of the following functions. Find the inverses and write your answers in $y =$ form.

a. $y = \log_9(x)$

b. $y = 10^x$

c. $y = \log_6(x + 1)$

d. $y = 5^{2x}$

5-72. Practice your logarithm fluency by calculating each of the following, *without changing the expressions to exponential form*. Be ready to explain your thinking.

a. $\log_7 49 =$ ____

b. $\log_3 81 =$ ____

c. $\log_5 5^7 =$ ____

d. $\log_{10} 10^{1.2} =$ ____

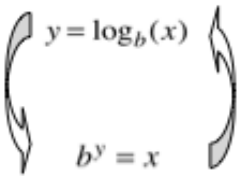
e. $\log_2 2^{w+3} =$ ____

METHODS AND MEANINGS

MATH NOTES

Logarithms and Their Notation

A **logarithm** (called a "Log" for short) is an exponent. An expression in logarithmic form, such as $\log_2(32)$, is read, "*the log, base 2, of 32*." To evaluate log expressions, think of the exponent: $\log_2(32) = 5$, because the exponent needed for base 2 to become 32 is 5.



An equation in logarithmic form is equivalent to another equation in exponential form, as shown below. This conversion helps show why (based on an $x \rightarrow y$ interchange) $y = \log_b(x)$ and $y = b^x$ are inverse functions.

Review & Preview

5-73. Let $y = \log_2(x)$. Rewrite the equation so that it begins with $x =$. Think about how you defined $y = \log_2(x)$ if you get stuck. Put a large box around both equations. Do the two equations look the same? Do the two equations mean the same thing? Are they equivalent? How do you know? This is very important. Think about it, and write a clear explanation. Help (Html5)⇔Help (Java)

5-74. Every exponential equation has an equivalent logarithmic form and every logarithmic equation has an equivalent exponential form. For example,

$$\underset{\text{base}}{4}^{\overset{\text{exponent}}{3}} = 64 \quad \text{is equivalent to} \quad 3 = \log_{\underset{\text{base}}{4}} 64$$

exponent

Copy the table shown below and fill in the missing form in each row. Help (Html5)⇔Help (Java)

	Exponential Form	Logarithmic Form
a.	$y = 5^x$	
b.		$y = \log_7(x)$
c.	$8^x = y$	
d.	$A^K = C$	
e.		$K = \log_A(C)$
f.		$\log_{1/2}(K) = N$

5-75. Suppose you want to buy sugar. Packages of different sizes cost different amounts, but the relationship is not always proportional. That is, a bag twice as big does not usually cost twice as much. The chart shows the prices for various sizes of bags of sugar. Help (Html5)⇔Help (Java)

½ lb bag	\$0.95
1 lb bag	\$1.38
2 lb bag	\$1.92
5 lb bag	\$4.70
10 lb bag	\$9.04
20 lb bag	\$17.52

a. Find the rates in cost per pound. (Stores refer to this as unit pricing.)

b. Does the unit price increase or decrease with the size of the bag?

c. Does the unit rate change more drastically for smaller sizes or for larger sizes?

5-76. Although the quadratic formula always works as a strategy to solve quadratic equations, for many problems it is not the most efficient method. Sometimes it is faster to factor or complete the square or even just "out-think" the problem. For each equation below, choose the method you think is most efficient to solve the equation and explain your reason. Then solve the problems that can be factored. Help (Html5)⇔Help (Java)

a. $x^2 + 7x - 8 = 0$

b. $(x + 2)^2 = 49$

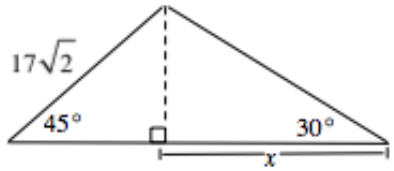
c. $5x^2 - x - 7 = 0$

d. $x^2 + 4x = -1$

5-77. If $10^{3x} = 10^{(x-8)}$, solve for x. Show that your solution works by checking your answer. Help (Html5)⇔Help (Java)

5-78. Find the value of x in each diagram below. Help (Html5)⇔Help (Java)

a.



b.

5-79. Consider the function defined by inputs that are the lengths of the radii of a circle, and outputs that are the areas of those circles. Write the equation for this function and investigate it completely. Help (Html5)⇔Help (Java)

5-80. Consider the equation $y = (x + 6)^2 - 7$. Help (Html5)⇔Help (Java)

a. Explain completely how to get a good sketch of the graph of $y = (x + 6)^2 - 7$.

b. Explain how to change the graph from part (a) to represent the graph of $y = (x + 6)^2 + 2$.

c. Given the original graph, how can you get the graph of $y = \left|(x+6)^2 - 7\right|$?

d. Restrict the domain of the original parabola to $x \geq -6$ and graph its inverse function.

e. What would be the equation for the inverse function if you restricted the domain to $x \geq -6$?