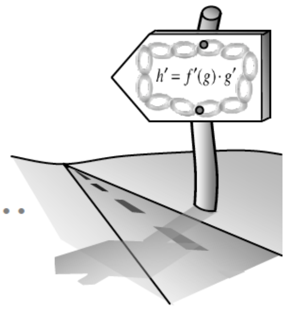
5.2.3 Can I use the Product Rule with the Chain Rule?

Chain Rule and Application: Part II



5-76. Differentiate each of these functions. (No need to simplify!) Use your calculators to check your solution.

a. $f(x) = (2x^4 - 1)^3$

b. $f(x) = \sin(2x^4 - 1)$

c. $f(x) = (\sqrt{x} + 3)^2$

d. $f(x) = (x^2 - 5x + 2)^4$

e. $f(x) = (x - 3)^2 + (x^2 + 1)^2$

5-77. The Chain Rule can be combined with other rules, such as the Product Rule, the rule for sums, and the rule for multiplying by constants. Differentiate these:

a. $f(x) = 5(2x^4 - 1)^3$

b. $f(x) = (2x^4 - 1)^3 + \sin(2x^4 - 1)$

c. $h(x) = x(2x^4 - 1)^3$

d. $f(x) = 3x \sin(2x^4 - 1)$

e. $f(x) = \sqrt{(x - 3)^2 + (x^2 + 1)^2}$

5-78. THE BUCKLED FREEWAY RETURNS!
Remember our buckled freeway problem (5-60)? The buckled freeway is an example of a composite function: the pavement's length depends on temperature, and temperature in turn depends on time of day.

a. Assuming that $t = 0$ is at midnight and is measured in hours, the temperature F, in degrees Fahrenheit, at time t is

$$F(t) = 20 \cos(0.25t - 4) + 85,$$

while the length of the portion of freeway (in meters) for temperature F is

$L(F) = 0.0008F^2 + 238.9.$

Write an equation relating the length L with time t.

b. Find $\frac{dL}{dt}$. What does $\frac{dL}{dt}$ represent?

c. During the day, when did this portion of the road reach its maximum length? What was the length?

5-79. Sometimes people make mistakes when differentiating using the Chain Rule. Explain the mistakes in the work below. Then write the correct derivatives. Homework Help ✎

a. $f(x) = -5(6x^4 - 1)^{10} + \pi$
$f'(x) = -50(24x^3 - 1)^9$

b. $f(x) = \sin(x^2 - x)$
$f'(x) = -\cos(x^2 - x)$

5-80. For parts (a) and (b) below, explain why finding the derivative requires the use of the Chain Rule and the Product Rule. Then find the derivative. Homework Help ✎

a. $\frac{d}{dx}\sqrt{\sin x \cos x}$

b. $\frac{d}{dx}\left(\sqrt{x}\,\sin(x^3)\right)$

5-81. If the height (in meters) of an object traveling vertically is represented by the function $h(t) = -4.9t^2 + 21t + 3$ where t is measured in seconds, find its: Homework Help ✎

a. Starting velocity

b. Starting position

c. Maximum height

5-82. FUNCTIONS FROM TABLES, Part One
Find the function $f(x)$ that fits the table shown below. What kind of function is it? Homework Help ✎

x	$f(x)$
0	1
1	5
2	9
3	13
⋮	⋮

5-83. Recall that $\sin^3 x = (\sin x)^3$. Find $\frac{d}{dx}(\sin^3 x)$. Homework Help ✎

5-84. Evaluate the following integrals *without a calculator*. Homework Help ✎

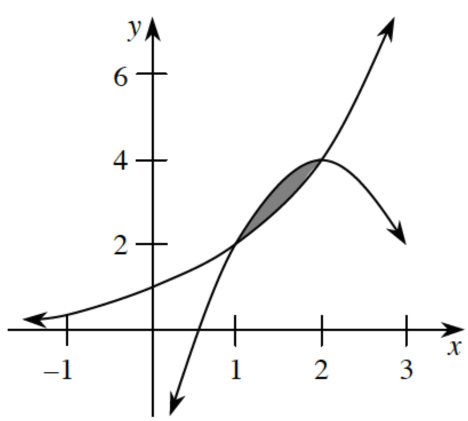
a. $\int \cos(5x)\,dx$

b. $\int_1^4 \frac{5}{\sqrt{x}}\,dx$

c. $\int_{-2}^{1} \frac{24}{\sqrt{x+3}}\,dx$

d. $\int (9x+4x^3)\,dx$

5-85. Use your calculator to estimate the shaded area shown between the graphs of $4-2(x-2)^2$ and $y = 2^x$. Homework Help ✎



5-86. Algebraically determine where $g(x)$ below is increasing, decreasing, concave up, and concave down. Homework Help ✎

$$g(x) = x^3 - 2x^2 + x - 1$$