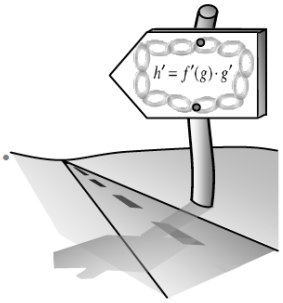
5.2.4 How do I differentiate a quotient?



Quotient Rule: Two Proofs

5-87. Now you can differentiate *powers* and *products*. In this problem, you will discover how to use these techniques on *quotients*.

a. Explain why $\frac{1}{2x+1} = (2x+1)^{-1}$. Then use this fact to find $\frac{d}{dx}\left[\frac{1}{2x+1}\right]$.

b. Now find $\frac{d}{dx}\left[\frac{x}{2x+1}\right]$ by first rewriting $\frac{x}{2x+1}$ as a product. Simplify your result.

5-88. Rewrite each of these as a product and then find the derivative. Use the Product Rule only if you need to.

a. $y = \frac{\sqrt{16-x^2}}{3}$

b. $f(x) = \frac{5x}{(2x+1)^3}$

5-89. Since many functions are written in the form $j(x) = \frac{f(x)}{g(x)}$, it would be convenient to derive a general formula for the derivative of quotients. Below are two demonstrations of how the **Quotient Rule** can be derived. Work through each demonstration and then compare the results. Are they the same?

Leslie's Proof	Tom's Proof
1. Rewrite $\frac{f(x)}{g(x)}$ as a product.	1. Let $j(x) = \frac{f(x)}{g(x)}$ and solve for $f(x)$.
2. Use the Product Rule and the Chain Rule to find $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$.	2. Use the Product Rule to find $f'(x)$.
3. Simplify as much as possible.	3. Solve for $j'(x)$.
	4. Simplify as much as possible.

5-90. Use your Quotient Rule to find y' if:

a. $y = \frac{\sin(x)}{\sqrt{x}}$

b. $y = \frac{3x^2-2x+1}{5x-1}$

c. $y = \frac{5-6x^2+2x^3}{x-x^2}$

d. $y = \frac{1}{x^3} + \frac{1}{x^4}$

5-91. Sometimes the Quotient Rule is not your only choice of differentiation techniques. Find $\frac{d}{dx}\left(\frac{3}{x-2}\right)$ in two ways: using the Quotient Rule and using the technique from problem 5-87. Which method is easier?

MATH NOTES

The Quotient Rule

The Quotient Rule states that if $h(x) = \frac{f(x)}{g(x)}$, then $h'(x) = \frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$.

Refer to problem 5-89 for two proofs of this result using the Product Rule and the Chain Rule.

5-92. THE CONTRIVED FUNCTION COMPANY
The contrived function company has just opened and wants to keep track of their resource-cost ratio. The resources are determined by the amount of money available. According to company projections, the anticipated resources function is $r(x) = (x - 3)^2 + 8$, where x is in months and $r(x)$ is in thousands of dollars. The cost function is $c(x) = 2x + 1$.
Calculate when the company will have its lowest resource-cost ratio, $\frac{r(x)}{c(x)}$.

Review & Preview

5-93. Suppose you are given a function $f(x) = x^3 + x^{2/3}$. Determine all critical points. Which points are minimums, maximums, and points of inflection? Homework Help ✎

5-94. Use your derivative tools to differentiate each function. Homework Help ✎

a. $\frac{d}{dx}\left(\sin \sqrt[3]{x+3}\right)$

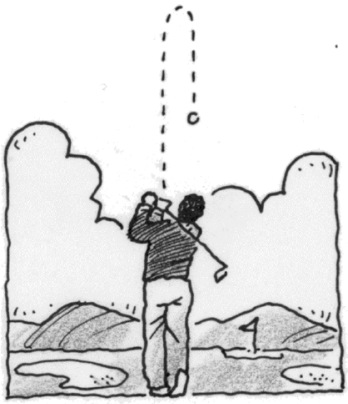
b. $\frac{d}{dx}\left(\frac{4-x^2}{x^3-2}\right)$

c. $\frac{d}{dx}\left(\frac{1}{\cos x}\right)$

d. $\frac{d}{dx}\left((6x-1)\sin(2x)\right)$

e. $\frac{d}{dx}\left(\frac{\sin(3x)}{2x^3}\right)$

f. $\frac{d}{dx}\left(\sqrt{\sin^2 x+\cos^2 x}\right)$



5-95. When Regit hit his golf ball at the 18th hole, it went straight up in the air! Homework Help ✎

a. If he hit it with an initial velocity of 144 feet per second, write an equation for the ball's velocity, $v(t)$, at time t. Assume the gravitational constant $a(t) = -32$ ft/sec^2 and that Regit hit the ball while it was on the ground.

b. When was the ball at rest? What is happening at that point in time?

c. What was the maximum height of the ball?

5-96. Simplify the following *without a calculator*. Homework Help ✎

a. $\int_1^8 \left(\sqrt[3]{27u}\right)du$

b. $\frac{d}{dx}\left(\int_4^{18}(6x-3)\,dx\right)$

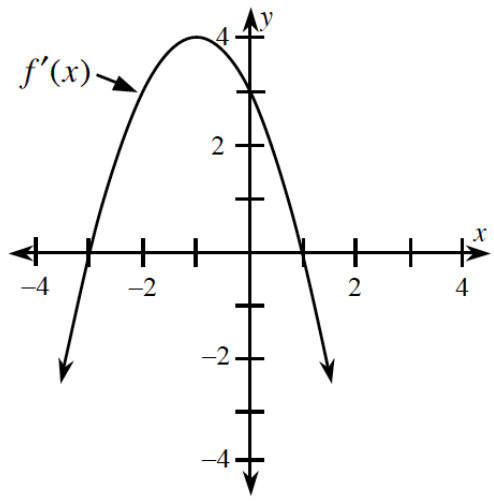
c. $\int f''(x)\,dx$

5-97. Sketch each of the following piecewise functions. Then, determine if the functions are continuous and differentiable over all reals. Homework Help ✎

a. $f(x)=\begin{cases} 2x^3 & \text{for } x<0 \\ 3x^2 & \text{for } x\geq 0 \end{cases}$

b. $f(x)=\begin{cases} (x+1)^2+1 & \text{for } x<-2 \\ |x| & \text{for } -2\leq x<2 \\ \sin(x-2)+2 & \text{for } x\geq 2 \end{cases}$

5-98. Using the graph of $f'(x)$, determine the values of x where $f(x)$ has a local minimum, local maximum or point of inflection. Justify your answer for each point. Homework Help ✎



5-99. For the following functions, find $f(x)$, c, $f'(x)$, and $f'(c)$. Homework Help

a. $f'(c) = \lim\limits_{\Delta x \to 0} \frac{(5-3(1+\Delta x))-2}{\Delta x}$

b. $f'(c) = \lim\limits_{\Delta x \to 0} \frac{(-2+\Delta x)^3+8}{\Delta x}$

c. $f'(c) = \lim\limits_{x \to 9} \frac{2x^{1/2}-6}{x-9}$