5.2.5 How can I build a new function?

Investigating Compositions of Functions

Today you will work with your team to create and analyze new, interesting functions that are compositions of functions with which you are already familiar.

5-105. Polly Parabola's first corporate venture, Professional Parabola Productions, was so successful that Felix's Famous Functions bought her out in a corporate takeover. With all of the money she made from the transaction, she has decided to start a new company, Creative Compositions. Creative Compositions plans to develop a line of composite functions designed to appeal to the imagination of the next generation of function groupies. She wants to market three new functions and is offering huge contracts to the winners of the competition. Your boss wants your company to enter this competition and has assigned your team to the development department.

CREATIVE COMPOSITIONS

Call for new and visually interesting compositions of functions

The Creative Composition Corporation announces an open competition for contracts to design new products. The products must be a composition of two or more functions whose parent functions are listed below:

$f(x) = x^2$ $g(x) = x^3$ $h(x) = 2^x$ $i(x) = \frac{1}{x}$ $j(x) = \sqrt{x}$ $k(x) = |x|$ $l(x) = \log_3 x$

Competing teams will prepare a poster to display their composite function and respond to questions from a panel of judges. Three contracts will be awarded based on the evaluation of the judges.

The judges will base their review on the following:

Is the graph of the composition a new and interesting shape?

Are multiple representations used effectively to show key features of the new function?

Does the selection of examples show off a variety of ways the function will appear when it is transformed?

Your Task: With your team, try out different ways to write compositions involving two or more of the given functions and check their graphs. Record everything you try as documentation for the report you will need to give your boss. When your team agrees on a function they like, investigate it thoroughly and prepare a poster for the competition.

Discussion Points

What does the graph of each function look like separately?

How does making the output of one function the input of the other change the original graph?

How do we have to adjust the domains and ranges?

Is the inverse a function?

Further Guidance

5-106. Consider $h(x) = 2^x$ and $k(x) = |x|$. Write the equation for each composite function $k(h(x))$ and $h(k(x))$. Discuss what each graph will look like and then sketch it. For each graph, explain the effect of one parent function on the other.

5-107. Choose other pairs of parent functions from the list. Then write the composite functions in both directions. In other words, use one function as the input for the other and then switch. Check the graphs and decide whether either is a good candidate for the competition. Try out at least five different pairs and record your equations and sketches of their graphs.

5-108. As a team, decide which of the functions you created that you want to enter in the competition. Now do a thorough investigation of that function.

5-109. Prepare a poster to show off your new function. Be sure to include all of the important details from your investigation on your poster and be prepared to respond to the judges with your arguments for why this function should be selected as one of the new products of Creative Compositions.

Further Guidance section ends here.

5-110. Consider the functions $f(x)$ in parts (a) and (b) below. For each $f(x)$, find two functions $h(x)$ and $g(x)$, so that $h(g(x)) = f(x)$. Use numerical examples to demonstrate that your functions $h(x)$ and $g(x)$ work.

a. $f(x) = \sqrt{3x+6}$

b. $f(x) = \frac{5}{\sqrt{x}}$

c. **Additional Challenge:** Work with your team to find another possibility for $h(x)$ and $g(x)$ such that $h(g(x)) = f(x)$ for each function given in parts (a) and (b). Be prepared to share your ideas with the class.

5-111. LEARNING LOG

Create a Learning Log entry explaining what you have learned about compositions of functions. Use examples to illustrate your ideas. Title this entry "Compositions of Functions" and label it with today's date.

Review & Preview

5-112. If $f(x) = \sqrt{7-x} - 6$ and $g(x) = -(x+6)^2 + 7$, find $f(g(x))$ and $g(f(x))$. What do the results tell you about $f(x)$ and $g(x)$? Help (Html5)⇔Help (Java)

5-113. For functions of the form $f(x) = mx$, it is true that $f(a) + f(b) = f(a + b)$. For example, when $f(x) = 5x$, $f(a) + f(b) = 5a + 5b = 5(a + b)$ and $f(a + b) = 5(a + b)$. Is $f(a) + f(b) = f(a + b)$ true for all linear functions? Explain why or show why not. Help (Html5)⇔Help (Java)

5-114. Find the value of x in the equation $2^x = 3$. Make your answer accurate to three decimal places. Help (Html5)⇔Help (Java)

5-115. Consider the following three sequences: Help (Html5)⇔Help (Java)

$$t(n) = 50 - 7n \qquad h(n) = 4 \cdot 3^n \qquad q(n) = n^2 - 6n + 17$$

a. Which, if any, is arithmetic? Geometric? Neither?

b. Are there any terms that all three sequences have in common? Justify how you know for sure.

c. Are there any terms that two of them share? Justify how you know for sure.

5-116. Using the sequences in the previous problem, suppose we define a new sequence, $s(n)$, defined as $s(n) = q(t(n))$, a compostion of two sequences. Do you think the new sequence will be arithmetic? Geometric? Neither? Explain. Make a table of values. Does the table support your hypothesis, or do you want to change your guess? Explain. Help (Html5)⇔Help (Java)

5-117. Gary has his function $g(x) = 10^x$ and Amy has her function $a(m) = 10^m$. Help (Html5)⇔Help (Java)

a. Each person is going to choose a whole number at random from the numbers 1, 2, 3…10, and substitute it into his or her respective

function. After they do this, what is the probability that $g(x) = a(m)$?

b. Find and simplify an expression for $g(x) \cdot a(m)$.

5-118. Sketch the graph of $y = 3\log(x + 4) - 1$. Help (Html5)⇔Help (Java)

5-119. Solve the system of equations below. Help (Html5)⇔Help (Java)

$$x + y = -3$$
$$2x - y = -6$$
$$3x - 2y + 5z = 16$$

5-120. Solve for m: $m^5 = 50$. Help (Html5)⇔Help (Java)

5-121. Consider two functions $f(x) = \log x$ and $g(x) = |x|$. Help (Html5)⇔Help (Java)

a. Use these two functions to write an equation for a composite function and sketch its graph.

b. Use these two functions to write a different composite function and sketch its graph.

c. What makes the two composite functions so different from each other?

d. **Challenge:** Now try graphing $g(f(g(x)))$.

5-122. Sketch square $ABCD$ on your paper, then randomly choose a point on $\overline{AB}$ and label it X. Draw $\overline{XC}$ and $\overline{XD}$ to form ΔXCD. If a dart is thrown and lands inside the square, what is the probability that it landed inside ΔXCD? Does it matter where you place X on $\overline{AB}$? Help (Html5)⇔Help (Java)

5-123. Solve $5^x = 15$ for x. Be accurate to two decimal places. Help (Html5)⇔Help (Java)

5-124. Some of the following algebraic fractions have common denominators and some do not. Add or subtract the expressions and, if possible, simplify. Help (Html5)⇔Help (Java)

a. $\frac{3}{(x-4)(x+1)} + \frac{6}{x+1}$

b. $\frac{5}{2(x-5)} + \frac{3x}{x-5}$

c. $\frac{x}{x^2-x-2} - \frac{2}{x^2-x-2}$

d. $\frac{x+2}{x^2-9} - \frac{1}{x+3}$

5-125. Simplify each of the expressions in parts (a) through (c) below. Help (Html5)⇔Help (Java)

a. $ab(\frac{1}{a} + \frac{1}{b})$

b. $cd(\frac{3}{c} + \frac{2c}{d})$

c. $x(1 - \frac{1}{x})$

d. What expression would go in the box in order to make the equation $\square\left(\frac{5}{x} + \frac{8}{y}\right) = 5y + 8x$ true?