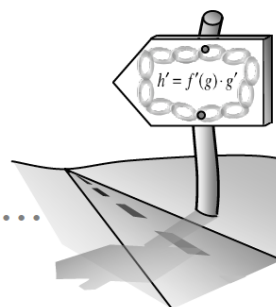


5.2.5 What are the derivatives of the other trig functions?

More Trigonometric Derivatives: $\tan x$, $\cot x$, $\sec x$, and $\csc x$



5-100. Which of the functions below are equivalent? Which are not? Explain.

- i. $\sin^{-1} x$
- ii. $\csc x$
- iii. $\arcsin x$

5-101. MORE TRIGONOMETRIC DERIVATIVES

Now that you know the Quotient Rule, you can differentiate the other trigonometric functions:

$$y = \tan x \quad y = \cot x \quad y = \sec x \quad y = \csc x$$

To do this, you will need to use the trigonometric identities.

- a. Rewrite $\tan x$ as a ratio of sine and cosine. Then find $\frac{d}{dx}(\tan x)$.
- b. Prove that $\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$. Find a similar formula for $\frac{d}{dx}(\csc x)$ and $\frac{d}{dx}(\cot x)$.
- c. Organize your derivatives into a chart like the one below.

Trig Function	Derivative
$y = \sin x$	
$y = \tan x$	
$y = \sec x$	

Trig “Co-Function”	Derivative
$y = \cos x$	
$y = \cot x$	
$y = \csc x$	

- d. Look for a pattern that will help you remember these derivatives.

5-102. A MIXTURE OF DERIVATIVE PROBLEMS

Using the properties you have learned, find the derivative of each of the following:

- a. $f(x) = \frac{\sin x}{x}$
- b. $f(x) = x^2 \cos(\sqrt{x})$

c. $f(x) = \tan(x^2)$

d. $f(x) = \sqrt{\frac{x^2+1}{x}}$

e. $f(x) = \frac{1}{1+\frac{1}{x}}$

f. $f(x) = \sec x \csc x$

g. $f(x) = \cos^3\left(\frac{x}{x+1}\right)$

h. $f(x) = \cot\left(\sqrt{\cos x + 1}\right)$



5-103. FUNKY FUNCTIONS REVISITED [Homework Help](#)

- Graph $f(x) = |x^3 + 0.125|$ and rewrite $f(x)$ as a piecewise function.
- Zoom in at $x = -0.5$ on your graphing calculator and carefully examine the curve. Does $f(x)$ appear differentiable at $x = -0.5$? Why or why not?
- To confirm whether or not $f(x) = |x^3 + 0.125|$ is differentiable at $x = -0.5$, we need to examine $f'(x)$. Use the piecewise function from part (a) to find $f'(x)$ for $x \neq -0.5$.
- Does $\lim_{h \rightarrow 0^-} \frac{f(-0.5+h) - f(-0.5)}{h} = \lim_{h \rightarrow 0^+} \frac{f(-0.5+h) - f(-0.5)}{h}$? State a conclusion.

5-104. Let $f(x) = x^{1/3}$. [Homework Help](#)

- Use the formal definition of continuity to determine if $f(x)$ is continuous at $x = 0$.
- Calculate $\lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x - 0}$.
- What does the limit in part (b) represent?
- What is happening graphically on $f(x)$ at $x = 0$ that causes your answer to part (b)?

5-105. You can use a tangent line to estimate the value of a function at points near the point of tangency. [Homework Help](#)

- Sketch a graph of $y = \sqrt{x+1}$ and its tangent line at $x = 0$. Then find the equation of this tangent line.

b. Use the tangent to approximate $\sqrt{x+1}$ for $x = 0.1$ and $x = -0.1$.

c. Use your calculator to evaluate $\sqrt{1.1}$ and $\sqrt{0.9}$. Find the percentage error for your estimate in each case. How accurate was your approximation?

5-106. Use your derivative tools to find the second derivative, $\frac{d^2y}{dx^2}$, of each function below. [Homework](#)

[Help](#) 

a. $y = \frac{\sin x}{x}$

b. $y = \csc^2 x - \cot^2 x$

c. $y = \sqrt{\frac{1}{x}}$

d. $y = |x - 2|$

5-107. Examine the integrals below. Consider the multiple tools available for evaluating integrals and use the best strategy for each. After evaluating the integral, write a short description of your method.

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a. $\int_{-3}^2 (-|x+1| + 2) dx$

b. $\int \left(\frac{2}{t^2} - t^3 \right) dt$

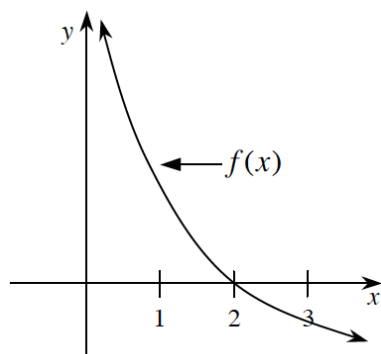
c. $\int_5^5 \log(\sqrt{2u^5 + 1}) du$

d. $\int \sec^2(x) dx$

5-108. Put the following in order from least to greatest given the graph of $f(x)$ below: [Homework Help](#)



$f(2)$, $f'(2)$, and $f''(2)$



5-109. A flag is formed by the region bounded by $y \leq x + 3$, $y \leq -2x + 12$, and the x -axis. Find the volume of the solid formed by rotating the flag about the x -axis. [Homework Help](#) 