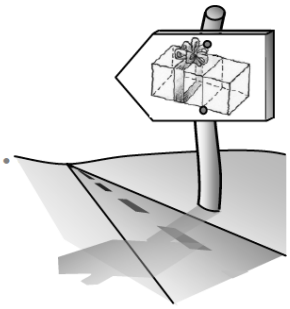
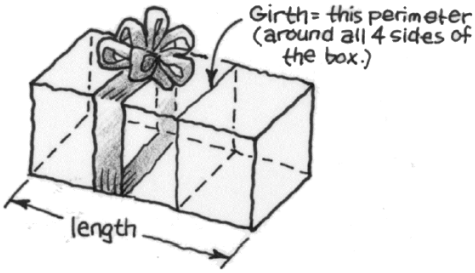
5.3.1 What is the shortest path?



Optimization Problems: Part I

5-110. THE POST OFFICE, Part One



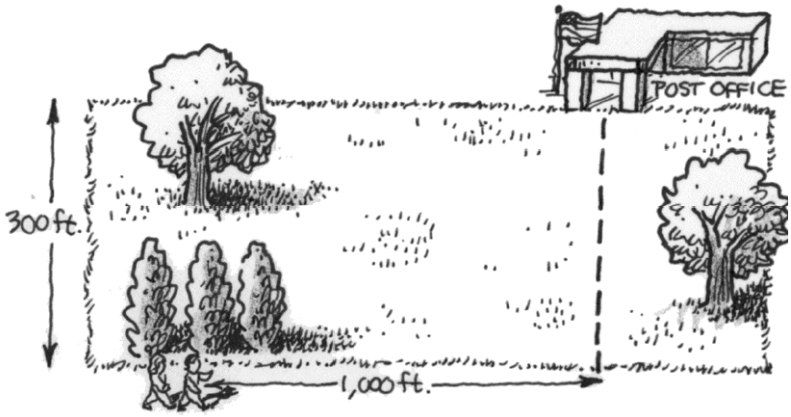
All calculus classes are required to mail their teacher a box of candy for Valentine's Day. Whichever class sends the most candy wins! However, the Post Office will only accept boxes where the sum of the length (the longest dimension) and the girth (the perimeter of a cross-section perpendicular to the length) is 108 inches or less.

a. The last class was trying to choose between two different boxes: a box full of Gummi Geckos that measured 29"x 20"x 20", and a box of Tastee Toads that was 30"x 19"x 19" where length is the first dimension. Which should they choose, and why?

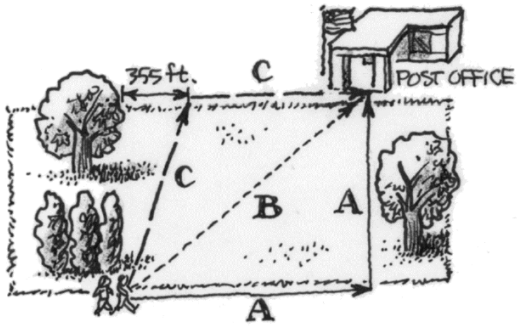
b. Assuming the box has *square* ends, find the dimensions of the box that holds the greatest volume of candy while still meeting the Post Office's size limits.

5-111. THE POST OFFICE, Part Two

Carrie and Tamana are running to take their teacher's candy box to the Post Office before it closes. The Post Office is on the other side of a grassy park, 1,000 feet east and 300 feet north of where they are.



They start to argue about how to get to the Post Office the fastest. If they run along the edge of the park, they can move at 6 feet per second but if they run through the grass, they can only run at 4 feet per second (lots of people walk their dogs in the park, so the two students have to be careful where they step).



Path A - Sidewalk and then straight across.
Path B - Straight across the grass.
Path C - A combination.

a. Above are three possible paths they could take. Find the total amount of time each path would take.

b. The Post Office is about to close, so every second counts! Find the path that would allow Carrie and Tamana to get to the Post Office in the least amount of time, and state the amount of time it will take.

Review & Preview

5-112. Differentiate the functions. Homework Help

a. $y = \frac{-2}{x^3}$

b. $y = \tan(x^2 + 3)$

c. $y = -5(x - 3)^8$

d. $g(x) = \frac{\cos x}{(x-3)^8}$

e. $y = x \tan(x^2 + 3)$

f. $y = \frac{3}{\cos t}$

5-113. FUNCTIONS FROM TABLES, Part Two

Find a function, $f(x)$, that fits the table shown below. Homework Help

x	$f(x)$
0	1
1	2
2	4
3	8
4	16
:	:

a. What kind of function does this appear to be? Find a possible equation.

b. How does the function change? Find the difference between the terms in $f(x)$, what kind of function is represented by these terms?

c. Find $f(5)$. Are you sure you are correct?

5-114. Carol threw an eraser straight up with a starting velocity of 12 m/sec and a starting height of one meter. Homework Help ✎

a. How high was the eraser after 1 second? After 2 seconds?

b. When did the eraser reach its maximum height? What was the maximum height?

5-115. Ms. Platinum has a hit on her hands. A survey she commissioned showed that she could sell 100 seats to her smashing new musical for \$40 each, and for each \$1 she drops the price, she could sell 10 more seats. Too bad the theater she rented only seats 200! How much should she charge for tickets to maximize her revenue? Homework Help ✎

5-116. When interest is compounded *quarterly*, the annual percentage rate is split into *four* equal parts and distributed four times during the year, at equally spaced time intervals. Compounding yields more money, because you get interest on previous interest. The easiest way to compute your new amount of money after an interest period is by multiplying the old amount by a suitable number, which we will call the **multiplier.** What multiplier applies to each of these situations? (Give both the unsimplified form and the simplified form.)

Example: 6% annual interest compounded quarterly.

Solution: *Unsimplified*: $1+\frac{0.06}{4}$. *Simplified:* 1.015. Homework Help ✎

a. 6% annual interest compounded twice a year.

b. 6% annual interest compounded monthly.

c. 6% annual interest compounded daily.

5-117. Examine the integrals below. Consider the multiple tools available for evaluating integrals and use the best strategy for each. After evaluating the integral, write a short description of your method. Homework Help ✎

a. $\int_{-2}^{1}(3x+1)^2\,dx$

b. $\int \csc^2(x)\,dx$

c. $\int_{x}^{0}\sin(t)\,dt$

d. $\int_0^{\pi} \cot(x)\tan(x)\,dx$