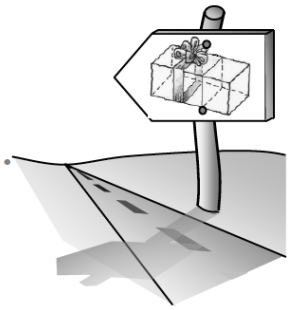
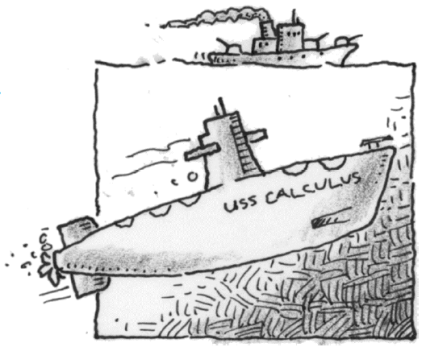
5.3.2 What is the minimum distance?



Optimization Problems: Part II



5-118. The U.S.S. Calculus, the United States' premier submarine, is in pursuit of the evil destroyer, Khaos. For navigational purposes, a navigator chooses an easily identifiable location to represent the origin and then finds a path according to this grid. The submarine is traveling on a path $y = x^2$ while the Khaos is stationary at the point (2, 0.5). The U.S.S. Calculus will wait until they are closest to the destroyer before launching their massive weapons of destruction.

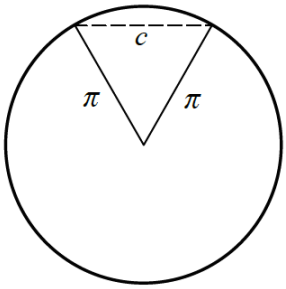
a. Sketch a graph to represent this scenario.

b. Find the location at which the submarine will be closest to the destroyer.

c. Find an equation of the line on which the torpedoes should travel.

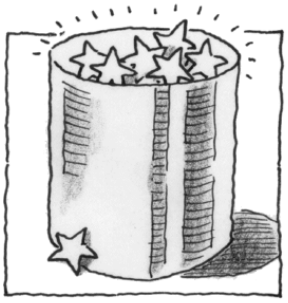
d. The line you found in part (c) above is*perpendicular* to the tangent lines and is called a **normal line**. Show that the line whose equation you found in part (c) above is in fact perpendicular to $y = x^2$ at (1, 1).

5-119. MORE PI DAY PIES



Several of the pies made for Pi day were overcooked and ended up with a burnt crust. Instead of throwing away the pies, the Math Club has decided to remove the burnt crust by cutting the pie pieces into isosceles triangles as shown in the diagram. The customer can specify the length of the cut, c, shown by the dotted line. If the customer gives the length of the cut that will produces the maximum sized piece, they will get the piece for free. The isosceles triangle has a vertex at the center of the circle and the radius, in celebration of Pi day, is of course π. The Math Club does not want to give away the pieces to just anyone, so they expect an exact answer and not just a decimal approximation.

5-120. THE GIFT



For her birthday, the calculus class has decided to get together and make Ms. Sardine a can (with no lid) to hold her little gold stars. However, since they are saving money for the AP test in May, they need to build it as cheaply as possible! Robert thinks he can make a circular base for roughly $1 per square inch, while the material for the curved walls is $2 per square inch. In order to hold enough gold stars, Lani has determined that the volume inside the can should be 60 cubic inches. Help find the dimensions (i.e. the radius and height)

for the cheapest can.

a. Since we want to minimize the cost, find a function C that calculates the total cost of the can.

b. Use the relationship between the radius and the height to change C (if necessary) so that it is defined by only the radius or the height. (Hint: Listen to Lani!)

c. Solve for the dimensions for the most cost-effective can. What is the cost?

5-121. Find the first and second derivative of $y = x + \cos x$ on $[-2, 2]$ and use these to find the maxima, minima, and points of inflection of in the given interval. Remember to check the endpoints!

Review & Preview

5-122. Find the derivative of each function. Homework Help

a. $f(x) = x\sqrt{5 - x^2}$

b. $g(x) = 2\cot(5x) - 2\cos^3 x$

5-123. Sketch an example of a graph with the given characteristics. Assume the graph is continuous and differentiable everywhere unless you are told otherwise. Comment on local or global maxima and minima. Find a suitable function for as many as you can. Homework Help

a. $f'(0) = f'(4) = 0$, $f''(0) > 0$, and $f''(4) < 0$.

b. $g(x)$ has only one critical point (at $x = 4$) and $g'(4) = 0$. $g'(x) < 0$ if $x < 4$ and $g'(x) > 0$ if $x > 4$.

c. Same as part (b), but $g'(4)$ is not defined.

d. $h(x)$ has a global minimum at $(0, 3)$, but the first and second derivatives are both zero there. Nevertheless, $h'(x) > 0$ if $x > 0$ and $h'(x) < 0$ if $x < 0$.

5-124. Suppose you deposit \$50 in Piggy Bank, which pays 5.5% annual interest. How much money will be in your account: Homework Help

a. After 6 months if the interest is compounded twice a year?

b. After 1 year if the interest is compounded twice a year?

c. After 1 year if the interest is compounded monthly?

d. After 2 years if the interest is compounded monthly?

e. After 2 years if the interest is compounded daily?

f. Why was your answer to part (e) larger than your answer to part (d)?

5-125. FUNCTIONS FROM TABLES, Part Three

Find the function, $f(x)$, that fits the table shown below. Homework Help ✎

x	$f(x)$
0	1
1	5
2	25
3	125
4	625
:	:

a. What kind of function does this appear to be?

b. How does the function change?

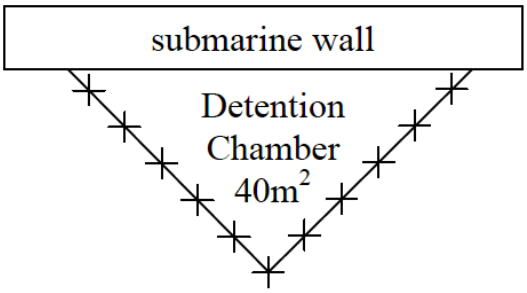
c. Do you know the value of $f(0.5)$?

5-126. Integrate. Homework Help ✎

a. $\int 2\sec(x)\tan(x)\,dx$

b. $\int \csc^2(2x)\,dx$

5-127. YELLOW SUBMARINE, Part Three



Ring's new three-chamber detention chamber was so successful that his friend John Lemon got jealous. Not to be outdone, John decided to make his chamber in the shape of an isosceles triangle. His chamber will still have 40 square meters of area. What dimensions should the chamber have so that Mr. Lemon can buy the least amount of barbed wire?
Homework Help