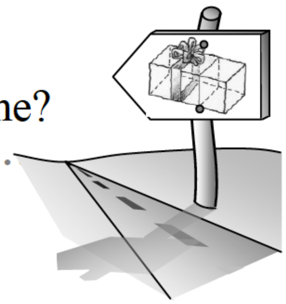
5.3.3 How can I maximize area and volume?



Optimization Problems: Part III

5-128. Shyrley makes pendants. She starts with a 10 cm length of wire and cuts it in two pieces, which she then forms into two geometric shapes.

a. Suppose one piece is bent into the perimeter of a square pendant and the other into the perimeter of an equilateral triangle pendant. How should she cut the wire in order to minimize the combined area of the two pendants?

b. Now suppose one piece is bent into the circumference of a circular pendant, and the other into the perimeter of a square pendant. Now how should she cut the wire in order to minimize the combined area of the two earrings?

c. Suppose Shyrley wants to maximize the combined area of the circle and square. Now how should she cut the wire?

5-129. MR. WAHMAN'S WHOPPER WAFFLE CONES!

Mr. Wahman loves to eat ice cream. He especially loves waffle cones, but he can never get the ice cream shop to make them the way he likes them. He wants cones which hold as much ice cream as possible, but without ice cream sticking over the edge of the cone (too messy). He decides to design his own waffle cones. His waffle iron makes circular waffles, which he can twist into a cone by first making one cut along a radius. He needs to know what size cone to make. What are the dimensions of the cone that holds the most ice cream (level to the top)?

Review & Preview

5-130. OLD SOFTWARE

$$y = 2^x$$
$$\log y = x \log 2$$
$$y = 10^{x \log 2}$$

Old computer programs used to only allow exponentials with a certain base (like your calculator only lets you use certain types of logs). If you needed an exponential in a different base, it had to be converted first. To the right is the procedure for changing a base 2 exponential function into base 10. Use this technique to change $y = 3^x$ into an exponential function with base 10.

5-131. WEI KIT RETURNS!

Wei Kit sure loves exponents! He's decided to rewrite all the numbers around him so that they have an exponent. For example, instead of writing the number 2, he writes:

$$3^{\log_3 2}$$

He insists he can also write the number 2 as $5^{\log_5 2}$ and $6^{\log_6 2}$! Homework Help ✎

a. Explain why Wei Kit's expressions all equal 2.

b. Use Wei Kit's method to rewrite the number 9 in two different ways.

c. Simplify this expression: $4^{(\log_4 5)x}$

d. How could Wei Kit rewrite 2^x so that it has a base of 7?

5-132. For each part below, what can you conclude (if anything) about $f(x)$ if you know the given information below? (Note: Each part is separate.) Homework Help ✎

a. $f'(-2) = 0$ and $f''(-2) > 0$.

b. $f'(x) < 0$ when $x > 3$, $f'(x) > 0$ when $x < 3$, and $f'(3) = 0$.

c. $f''(3) > 0$

d. $f''(3) < 0$

e. $f''(3) = 0$

f. $f(x)$ is continuous at $x = 3$, but not differentiable there.

g. $f(x)$ is defined and continuous everywhere, and has just one critical point at $x = 2$, which is a local maximum.

h. $f''(3)$ does not exist and $f''(x) < 0$ for $x < 3$ and $f''(x) > 0$ for $x > 3$.

5-133. When banks compound interest, they often advertise an **annual percentage rate.** This is the annual rate of simple (uncompounded) interest that would give you the same amount of money at the end of one year. Confirm that the annual percentage rate for 6% annual interest, compounded quarterly, is 6.14% (to the nearest 0.01%). Homework Help ✎

5-134. Find the equation of the line tangent to the graph of $y = \sec x$ at the point where $x = 2$. Include a sketch of the graph near $x = 2$ and the tangent line, clearly labeled. Homework Help ✎

5-135. If $y = (\cos x + \sin x)^2$, find $\frac{dy}{dx}$ using two different methods. Then demonstrate that the two results are equal. Homework Help ✎

5-136. Find the inverse function, $f^{-1}(x)$, for each of the following functions. Homework Help ✎

a. $f(x) = \log_3 x$

b. $f(x) = 6^{2x}$

c. $f(x) = \pi^2$

d. $f(x) = a^x + k$

5-137. Multiple Choice: $\frac{d}{dx}\int_0^{\pi/3} \sin(x)\,dx =$

Homework Help ✎

a. $\frac{\sqrt{3}}{2}$

b. $\frac{1}{2}$

c. 0

d. $-\frac{1}{2}$

e. undefined