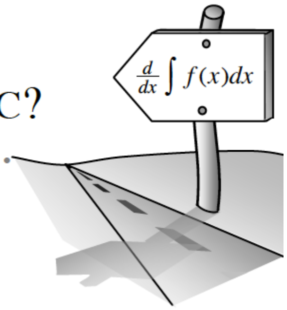
5.4.1 How is the Chain Rule applied to the FTC?



Chain Rule Extension of the Fundamental Theorem of Calculus

In Chapter 4 you learned the Fundamental Theorem of Calculus, which related derivatives to integrals. You now have the skills to apply your derivative rules to the Fundamental Theorem.

5-138. Complete the following.

a. Explain why the following equation is true.

$$\frac{d}{dx}\int_a^x f(x)dx = f(x)$$

b. Note that *x*, the upper bound is a function. Let's investigate a more complicated case $\frac{d}{dx}\int_a^{g(x)} f(t)dt$, by justifying the following steps.

i. $\frac{d}{dx}\int_a^{g(x)} f(t)dt$ — How is this integral different than the one in part (a)?

ii. $\frac{d}{dx}(F(g(x))-F(a))$ — What was done to get this?

iii. $f(g(x)) \cdot g'(x) - 0$ — What derivative rule was used?
Why did the second part become 0?

c. In general, $\frac{d}{dx}\int_a^{g(x)} f(t)dt =$ _____________

5-139. Using what you did in the previous problem, determine $\frac{d}{dx}\int_{h(x)}^{g(x)} f(t)dt = \ ?$

5-140. Use your new shortcuts to quickly evaluate each of the following expressions.

a. $\frac{d}{dx}\int_2^{x^2} \sin(t^5)dt$

b. $\frac{d}{dx}\int_3^{x^2} \sin(\csc(t))dt$

c. $\frac{d}{dx}\int_{\tan x}^{4} \cot(t^2)dt$

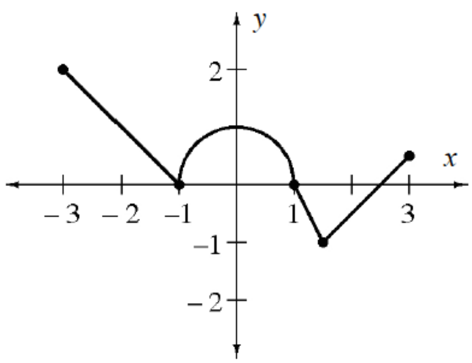
d. $\frac{d}{dx}\int_{\sin x}^{\cot x}\sqrt{1+t^3}\,dt$

e. $\frac{d}{dx}\int_{x^3}^{3\sin x}\sqrt{t^2-1}\,dt$

f. $\frac{d}{dx}\int_{\csc x}^{x^3}\frac{t-3}{t+1}dt$

Review & Preview

5-141. Given the graph of $f'(x)$ below, determine the values of x for which $f(x)$ has local minima, maxima, and points of inflection on the interval $[-3, 3]$. Homework Help ✎



5-142. Using the graph of $f'(x)$ in problem 5-141, and given that $f(0) = 2$, determine the following values. Homework Help ✎

a. $f(1)$

b. $f'(1)$

c. $f(-3)$

d. $f'(-2)$

e. $f''(2)$

5-143. Easy Limits

Evaluate each of the following limits. Homework Help ✎

a. $\lim\limits_{x\to 2}\frac{x^3-8}{x-2}$

b. $\lim\limits_{h\to 0}\frac{\sec(3(x+h))-\sec(3(x-h))}{2h}$

c. $\lim_{h \to 0} \frac{\cot\left(\frac{\pi}{4}+h\right)-1}{h}$

d. $\lim_{x \to \pi} \frac{\tan 5x}{x-\pi}$

e. What do all of the limits above have in common?

5-144. Find the coordinates for all of the relative minima, relative maxima and points of inflection for the function $f(x) = (x^2 - 1)(x + 1)^2$. Homework Help ✎

5-145. Find the equation of the line tangent to $y = \cot(2x)$ at $x = \frac{\pi}{3}$. Homework Help ✎

5-146. Multiple Choice: Let f be the function given by $f(x) = |x|$. Which of the following statements about f are true? Homework Help ✎

I. f is continuous at $x = 0$.
II. f is differentiable at $x = 0$.
III. f has an absolute minimum at $x = 0$.

a. I only

b. II only

c. III only

d. I and III

e. II and III

5-147. The function f is continuous on the closed interval [2, 8] and has values that are in the table to the right. Homework Help ✎

x	$f(x)$
2	10
5	30
7	40
8	20

a. Using the sub-intervals [2, 5], [5, 7] and [7, 8], what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$?

b. On what sub-interval is the average slope a minimum?

I) [2, 5] II) [5, 7] III) [7, 8]